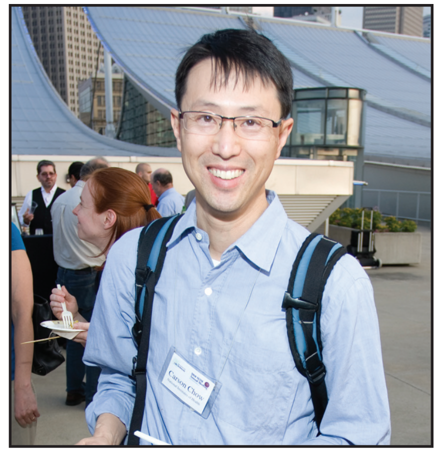
From *SIAM News*, Volume 43, Number 7, September 2010

Heavy Math Sheds Light on Weighty Issue

By Barry A. Cipra



Carson Chow gave a joint invited presentation on the dynamics of obesity at this year's SIAM Annual Meeting and Conference on the Life Sciences.

For mathematical obesity expert Carson Chow, the human body looks a lot like a leaky integrator. We can gain a lot, he says, by using such a bare-bones mathematical model to look at the problem of weight control. Much of the mystery and many of the myths surrounding Americans' expanding waistlines can be accounted for with a few simple differential equations. Unfortunately, the calculus of weight gain suggests that *solving* the problem may not be so simple.

Chow, a researcher in the laboratory of biological modeling at the National Institute of Diabetes and Digestive and Kidney Diseases, part of the National Institutes of Health, gave a joint invited presentation on the dynamics of obesity at this year's side-by-side SIAM Annual Meeting and Conference on the Life Sciences, held in Pittsburgh in July. Chow led the audience through the equations of macronutrient flux, body composition, and basal metabolism, to the implications of "life on the Forbes curve" for the relation between weight and weight gain.

The basic equations start in three dimensions: Weightwise, we worry about our intake of fat, carbs, and protein. (A good way to gain weight quickly is to drink a couple large glasses of water, but that's not the kind of weight gain anyone worries much about.) These nutrients are stored in the form of body fat F, glycogen G, and protein P; the latter two make up what's regarded as the body's "lean" mass L. The relevant equations simply relate the derivatives F', G', and P' to the difference between the (continuous) rate of intake (i.e., digestion) and the (continuous) rate of energy expenditure. These equations are then put on a reducing plan.

The first reduction eliminates G. The body doesn't store much glycogen, and on the time scales relevant for weight control, it's sensible to set $G' = 0$—the body, in other words, finds some way to dispose of all the calories that come in the form of carbohydrates, either by burning them or by converting the carbs to fat. This means that the dynamics can be recast in terms of F and L.

The next reduction comes courtesy of the late Gilbert Forbes, a pediatric nutritionist at the University of Rochester Medical Center. In a study of women and adolescent girls published in 1987, Forbes observed a logarithmic relation between fat and lean body mass: $L = 10.4 \ln(F) + 14.2$, in kilograms. The Forbes curve facilitates a discussion of the dynamics in the L–F phase plane. And if taken literally, it allows the whole discussion to boil down to a single differential equation, which can be cast in terms of body mass $M = L + F$. When properly linearized, the relevant equation has the form

$$\rho M' = I - b - \varepsilon M,$$

where I represents daily energy intake, b a baseline daily energy expenditure (which, on average, better be less than I—both being functions of time and the obvious "control" parameters in any diet and exercise program), ρ a conversion factor of calories into kilograms, and ε a factor that represents the effect of body mass on daily energy expenditure (e.g., the fact that a heavy person does more work going up a flight of stairs than someone who weighs less).

It's that ε that makes our integrators leak. (The leaky integrator is a staple of electrical circuits. The equation has also been borrowed by neuroscientists.) Without it, the linear approximation suggests that we'd either fatten without limit or waste away to nothing, depending on whether I exceeds b; with it, we gravitate toward a stable fixed point $M = (I - b)/\varepsilon$ (in the setting of constant diet and exercise).

The value of ε turns out to be approximately 10 Calories per pound per day. (A "Calorie," which is what you see listed on food labels, is a thousand "calories," or approximately 4200 joules. Nutritional notation makes it easy to be off by three orders of magnitude.) The other conversion factor, ρ, is the more familiar approximation, 3500 Calories per pound. Traditional popular diet literature has fixated on ρ. That's a mistake, Chow says: The more meaningful number is ε. If your goal is to drop a pound, you shouldn't think in terms of simply taking a week off from that humungous, 500-Calorie chocolate chip muffin you have each morning; you would do better to leave some sizable crumbs (i.e., about 2% of the muffin) on the plate *every* day.

By happenstance, the ratio $\rho/\varepsilon = 350$ days, roughly a year. This, not weeks or even months, is the time scale on which the effects of diet and exercise should be judged. Much as an apple a day keeps the doctor away, substituting an apple, at 100 Calories, for a 200-Calorie cookie will keep ten pounds at bay. The problem is, too many of us are eating both the apple and the cookie.

Barry A. Cipra is a mathematician and writer based in Northfield, Minnesota.