# **Brazilian Applied Math Targets Local Problems**

## By Iain Duff and José A. Cuminato

The expansion of the Brazilian economy in recent years has greatly increased the demand for new industrial research. Of particular interest are energyrelated research-from the petroleum industry (e.g., Petrobras), the sugar cane ethanol industry, the new hydroelectric plants being built in the Amazon basin-and aerospace research (from, for example, the aircraft manufacturer EMBRAER). To educate the increasing numbers of PhD scientists and engineers, including mathematicians and applied mathematicians, needed to perform the research, the university system has expanded over the past eight years. New research universities have been established, and the numbers of PhD programs in mathematics and applied mathematics have grown



The Brazilian Society for Computational and Applied Mathematics, founded in 1978, celebrated its 32nd anniversary on November 1, 2010. SBMAC has 600 members, has been a member society of ICIAM since its formation, and has membership agreements with many of its sister societies in Brazil and abroad.

The annual meeting of SBMAC-CNMAC (Brazilian Conference on Computational and Applied Mathematics)-is the main applied math meeting in Brazil. A huge meeting by any standards, it attracts about a thousand delegates each year, with a program of invited talks, work-

shops, mini-courses, submitted presentations, and poster sessions. One tradition of CNMAC is the substantial attendance of undergraduate students across many disciplines. The professional attendees represent both academia and industry, and work in mathematics, computer science, biology, medicine, engineering, and physics. The conference language is Portuguese, with special talks by well-known speakers sometimes given in English. The idea for this article originated at the conference held in September 2010 in Águas de Lindóia, near Campinas, where Iain Duff gave one such talk.

steadily. Today, 30 universities offer PhD programs in mathematics, applied mathematics, and statistics; about 150 PhDs are awarded each year. Even this growth, however, has not been enough to meet the increased demand from industry. Accordingly, the Brazilian Society for Computational and Applied Mathematics has been working with university departments to help improve the quality of new graduate courses and to develop industryrelated courses and activities, especially those concerned with mathematical modeling.

Evidence of the increasing importance of mathematics in industry can be seen in the recent launch of two initiatives: the Industrial Mathematics Institute (IMI), in the southern state of Paraná, and the Center for Mathematics and Statistics Applied to Industry (CeMEAI), a project of the University of São Paulo. IMI is a joint venture of the Center for the Industries of Paraná State (CIEP), a private association of the major industries in the state, and the Federal University of Paraná (UFPR). Inaugurated in December in Curitiba, IMI is housed in the headquarters of CIEP. CeMEAI will be located on the new campus of the University of São Paulo now being built in the city of São Carlos, in an area dedicated to industrial partnership laboratories.

#### **Main Industrial Problems**

To sustain its economic growth, Brazil is working to bring about a large increase in its energy production and output. Brazil is currently a world leader in clean energy consumption, with some 70% of the energy consumed derived from clean sources: Most of its electricity comes from hydropower plants, and most of its cars are bi-fuel, running on ethanol from sugar cane as well as on gasoline.

Several extensive projects are under way to increase Brazil's energy-generation capacity. Two large new dams are being built in the Amazon basin, the production of ethanol is increasing, and exploration of large new gas and oil fields discovered recently in the Atlantic Ocean will begin once the required new technologies have been developed.

The dams in the Amazon area, although designed to minimize flooding, will nevertheless lead to inundation of densely forested areas. Mathematical modeling will be important-models of the reservoirs to manage the flooding, and wildlife models to help with the rescue of animals and the study

of the migration of several species of fish important to the local economy. As to the goal of increasing ethanol production without incurring further deforestation, experts say that the way forward is through the use of degraded land previously used for grazing. According to S. Crestana, former chief executive of the Brazilian Institute for Agricultural Research, the modeling of land use is crucially important.

Pre-Salt Layer Oil Recovery. The oil and gas fields recently discovered in the Atlantic Ocean, some 800 kilometers offshore, present substantial new challenges to the Brazilian oil industry. In addition to the far-offshore location, the water depth in the area is about 2 kilometers and the oil and gas are buried very deep, about 7 kilometers under the sea bottom; the oil fields are below the salt layer, explaining their popular designation in Brazil as "presalt" fields (see Figure 1).

Of several new fields discovered, only one has been fully explored and modeled; its oil content is estimated to be 8 billion barrels. Petrobras plans to spend U.S.\$150 billion through the year 2025 to fully develop these oil fields. A substantial portion of these funds will be needed for research, and because the technology for oil exploration under such circumstances is not available, some of the resources will have to be directed to mathematical modeling.

Development of the new oil fields will require innovative computational Figure 1. Schematic illustration of offshore pre-salt oil reservoirs.



models capable of predicting complex coupled phenomena in highly heterogeneous geological formations. The scientific community thus faces the challenge of developing genuine multidisciplinary research, with fruitful collaboration among different groups, usually including applied mathematicians and geoscientists.

Preliminary analysis reveals the presence of varying amounts of carbon dioxide in several of the oil reservoirs in the pre-salt formations. The almost impermeable rock forming the saline cap can be exploited as a physical barrier to avoid leakage of  $CO_2$ , thus allowing for an efficient  $CO_2$  storage process. Such efficiency can also benefit enhanced light oil production, giving rise to highly sustainable technologies.

The appearance of strongly coupled phenomena of different natures in multiphase flows in such heterogeneous fractured and vuggy porous media, combined with the extensive variability and uncertainty in the rock properties, leads to complex regimes of fluid mixing. In addition to uncertain flow and transport with buoyancy effects governed by stochastic PDEs, the chemistry of the injected water with dissolved  $CO_2$  could strongly affect the wettability of the rock. The low pH of the aqueous solution, moreover, could trigger dissolution/precipitation reactions with the solid minerals; the resulting degraded rock stiffness would enhance reservoir compaction and surface subsidence.

Given these conditions, the stochastic model should also be capable of incorporating geochemical effects governed by equilibrium and non-equilibrium thermodynamics and their coupling with rock mechanics. The geomechanical coupling must incorporate chemical dam-

age resulting from the high water acidity, along with the viscoelastic behavior of the saline cap, which may undergo creep and become damaged, with the appearance of cracks and consequent deterioration of its sealing property. The crucial need here is to compute the optimal  $CO_2$  injection pressure in order to avoid fault reactivation; this would tend to increase fault permeability, with stress leading to preferential paths for  $CO_2$  migration and consequent loss of the efficiency of the  $CO_2$  sequestration.

In addition to the aforementioned features, the necessity of solving the problem in large domains with coarse meshes is connected to the use of multiscale or up-scaling methods constructed from underlying fine-scale properties. In particular, the fine-scale input properties can be obtained within geostatistical tools improved by history matching via, for instance, Bayesian statistics. All these issues need to be incorporated in a new generation of simulators; development of the underlying numerical models will require crucial input from the scientific community.

*Hydroelectric Dams: Biomass Evaluation and Flood Prediction.* The flooding of forested areas and the resulting decomposition of organic material deplete water of oxygen, killing fish and other underwater creatures. Cleaning up flooded areas is one of the requirements imposed by regulatory bodies. One solution, removal of all vegetation from the flooded area, is not feasible because of the extremely large volume and consequent cost. The modified policy is to remove vegetation from selected areas, leaving a biomass volume that will not pollute the water beyond a critical point. Many research groups in Brazil have collaborated on the design and refinement of a computer system for calculating the biomass in a reservoir. Using satellite pictures, maps, and GPS data col-



Figure 2. Two-dimensional mesh with depth.



Figure 3. Satellite pictures of a reservoir site before and after flooding.

lected on site, the system models the reservoir, generating a two-dimensional mesh that is extended to three dimensions with the addition of a small depth dimension (see Figure 2). The adaptive mesh is produced via Delaunay triangulation, and refined with a Delaunay refinement algorithm that respects prescribed level sets [1,2]. The transport equations are then solved on this mesh. Parameters are provided for the transport equations in two additional steps: identification of the vegetation and its localization on the mesh, and analysis of chemicals produced by the decomposing biomass. These two steps require the collaboration of other scientists, including geographers, ecologists, biologists, and biochemists. The mathematics in modeling this problem lies mainly in the mesh generation and in the solution of the transport equations on this mesh.

A computed simulation modeling the transport of the more important chemicals in the water predicts regions of the reservoir in which the water flow will not be strong enough to carry away the pollution (see Figure 3). It is in these regions that the vegetation should be cleared away. Because of the large scale differences (the surface dimensions of the reservoir are much larger than its depth), shallow water equations are used to simulate the flow in the reservoir [3]. The flooding of the reservoir can also be predicted, allowing for the planned rescue of animals and the logging of trees that are economically valuable.

## **SIAM in South America**

Brazil's neighbor Argentina founded its applied mathematics society in 2008. In fact, creation of the Asociación Argentina de Matemática Aplicada, Computacional e Industrial (ASAMACI, http://asamaci.unsl.edu.ar/) actually followed the formation (in 2006) of the Argentine Section of SIAM (see the accompanying report from AR–SIAM's third conference). AR–SIAM and ASAMACI, with many members in common, hold a biennial joint meeting called MACI (Congress on Industrial, Computational and Applied Mathematics). The third MACI was held May 9–11, 2011, in the city of Bahía Blanca, in Buenos Aires County. In even-numbered years, ASAMACI holds a one-day technical meeting. ASAMACI is in the process of becoming a member of ICIAM.

Colombia formed the Colombia Section of SIAM (COSIAM), the only other SIAM section in South America, in 2010, shortly after starting a SIAM student chapter at one of its major universities, Universidad Nacional de Colombia in Sede Manizales. COSIAM plans to hold local and regional meetings, in addition to an annual meeting. A major axis of these meetings is to be the interaction between academia and industry.

### **Acknowledgments**

We thank Marcio Murad of LNCC for collaboration on the pre-salt material and Antonio Castelo of the Gesar group for the material on the modeling of water dams.

#### References

[1] H.H. Bíscaro, A. Castelo, L.G. Nonato, and M.C.F. Oliveira, A topological approach for surface reconstruction from sample points, Vis. Comput., 23 (2007), 793–801.

[2] J.P. Gois, V. Polizelli, T. Etiene, E. Tejada, A. Castelo, L.G. Nonato, and T. Ertl, *Twofold adaptive partition of unity implicits*, Vis. Comput., 24 (2008), 1013–1023.

[3] N. Mangiavacchi, A.L. Coutinho, and N. Ebecken, *Turbulent shallow-water model for orographic subgrid-scale perturbations*, J. Braz. Soc. Mech. Sci., 22:1 (2000).

Iain Duff is a visiting professor in the Department of Mathematics and Statistics, University of Strathclyde, in Glasgow, UK, a CCLRC senior fellow in the Computational Science and Engineering Department at Rutherford Appleton Laboratory, in Oxfordshire, UK, and a scientific adviser at CERFACS, Toulouse, France. José A. Cuminato is a member of the Institute of Mathematics and Computer Sciences, São Paulo University, in São Carlos, Brazil, and a visiting professor in the Department of Mathematics and Statistics, University of Strathclyde.