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# Kinetic Modelling of Opinion Leadership

*By B. Düring*

In recent work my colleagues and I have introduced and discussed a nonlinear kinetic model for opinion formation [1] in a society made up of two groups, with strong opinion leaders in one and ordinary people in the other. The process of opinion formation has received growing attention from physicists, who have opened a research field termed *sociophysics* that dates back to the pioneering work of Galam et al. [3]. The literature on the subject has focused primarily on election processes, referenda, and public opinion tendencies. Less attention has been paid to the important effects of opinion leaders on the dissemination of new ideas and on the diffusion of beliefs in a society.

Opinion leadership is one of several sociological models developed to study the formation of opinions in a society. It builds on a two-step flow of communication, with so-called opinion leaders who are active media users selecting, interpreting, modifying, facilitating, and finally transmitting information from the media to less active parts of the population. Opinion leadership appears in settings that include but are not limited to political parties and movements, advertisement of commercial products, and dissemination of new technologies. Typical personal characteristics attributed to opinion leaders are high levels of confidence and self-esteem, and the ability to withstand criticism. Opinion leadership is specific to a subject and can change over time.

Our work is based on a kinetic model of opinion formation, introduced in [4]. It is built on two main aspects of opinion formation: In the first, a *compromise process*, individuals tend to reach a compromise after an exchange of opinions; in the second, *self-thinking*, individuals change their opinions in a diffusive way, possibly influenced by exogenous information sources like the media. Incorporating both, Toscani [4] introduced a kinetic model in which opinion is exchanged between individuals through pairwise *microscopic* interactions. Depending on the specification of these interactions, a certain *macroscopic* opinion distribution develops in the society at large. Mathematically, the model in [4] is related to work on the kinetic theory of granular gases. In particular, the non-local nature of the compromise process is analogous to the variable coefficient of restitution in inelastic collisions. Similar models have been used in the modelling of wealth and income distributions [2].

Opinion is represented as a continuous variable $w \in \mathcal{I}$ with $\mathcal{I} = [-1,1]$, where $\pm 1$ represent extreme opinions. In the context of political opinions, $\mathcal{I}$ can be identified with the left–right political spectrum. If two individuals with pre-interaction opinions $v$ and $w$, respectively, from the same group ($i = 1,2$) meet, their post-interaction opinions $v^*$ and $w^*$, respectively, in our model are defined, much as they are in [4], by

$$v^* = v - \gamma P_i(|v-w|)(v-w) + \eta_{i1} D_i(v), \quad (1a)$$
$$w^* = w - \gamma P_i(|w-v|)(w-v) + \eta_{i2} D_i(w). \quad (1b)$$

The sociophysical effect of opinion leaders on public opinion is based on the hypothesis that the leaders' opinions are not changed through interactions with regular society members. Therefore, when an ordinary person with opinion $v$ meets a strong opinion leader with opinion $w$, their post-interaction opinions are given by

$$v^* = v - \gamma P_3(|v-w|)(v-w) + \eta_{11} D_i(v), \quad (2a)$$
$$w^* = w. \quad (2b)$$

Here, $\gamma \in (0,1/2)$ is a constant *compromise parameter* that controls the "speed" of attraction of two different opinions. The quantities $\eta_{ij}$ are random variables with distribution $\theta$, variance $\sigma_{ij}^2$, and zero mean. The functions $P_i(\cdot)$ ($i = 1,2,3$) and $D_j(\cdot)$ ($j = 1,2$) model the local relevance of compromise and self-thinking, respectively, for a given opinion.

The first term on the right-hand side in (2a), (1a), and (1b) models the compromise process, the second the self-thinking process. Opinion leaders retain their opinions in (2b) when interacting with ordinary people, which reflects their high self-confidence and ability to withstand other opinions. In our model, opinion leaders can be influenced only through their peers, by interactions in (1). The pre-interaction opinion $v$ increases (gets closer to $w$) when $v < w$ and decreases in the opposite situation. We assume that the ability to find a compromise is linked to the distance between opinions: The greater this distance, the lower the possibility that a compromise will be found.Hence, functions $P_i(\cdot)$ are assumed to be decreasing functions of their argument. We also assume that the ability to change individual opinions by self-thinking decreases closer to the extremal opinions, reflecting the fact that extremal opinions are more difficult to change. Therefore, we assume that functions $D_j(\cdot)$ are decreasing functions of $v^2$, with $D_j(1) = 0$. In (1) and (2) we allow only interactions that guarantee $v^*, w^* \in \mathcal{I}$. To this end, we assume additionally that

$$0 \le P_i(|v - \mathrm{w}|) \le 1, \quad 0 \le D_j(v) \le 1,$$

and specify the range of values the random variables can assume.

In this setting, we are led to study the evolution of the distribution function for each group as a function depending on the opinion $w \in \mathcal{I}$ and time $t \in \mathbb{R}_+$, $f_i = f_i(w,t)$. In analogy with the classic kinetic theory of mixtures of rarefied gases, the time-evolution of the distributions will obey a system of two Boltzmann-like equations. The Boltzmann-like collision operators are derived by standard methods of kinetic theory, considering that the change in time of $f_i(w,t)$ resulting from binary interaction depends on a balance between the gain and loss of individuals with opinion $w$.

As is usual in kinetic theory, it is convenient to study certain asymptotics of this complex system, which frequently lead to simplified Fokker–Planck-type models. This approach makes it easier to identify steady states while retaining important information on the microscopic interaction at a macroscopic level. To this end, we study by formal asymptotics the quasi-invariant opinion limit ($\gamma, \sigma_{ij} \to 0$ with $\sigma_{ij}^2/\gamma = \lambda_{ij}$ kept fixed) following the path laid out in [4]. We derive a system of Fokker–Planck limit equations with non-local operators appearing in the drift term and $\lambda_{ij}$ as diffusion coefficients.

We analyze explicitly computable stationary states of the Fokker–Planck system for special choices of $P_i(|w-v|) \equiv 1$ ($i = 1,2,3$). This analysis, combined with numerical results for the general situation, strongly suggests that the Fokker–Planck system admits integrable stationary states in the general case as well, although they are not explicitly computable. Moreover, numerical simulations provide strong evidence that solutions converge exponentially fast to their steady state.

| | Grüne | SPÖ | ÖVP | FPÖ | BZÖ |
|---|---|---|---|---|---|
| 2004 | 6.7% | 38.4% | 11.6% | 42.5% | — |
| 2009 | 5.2% | 28.8% | 16.8% | 3.8% | 44.9% |

**Table 1.** *Results of state elections in Carinthia.*

Opinion leadership is not constant over time. Someone who is an opinion leader today could lose this role, or a follower could become a leader in the near future. The emergence and decline of opinion leaders thus constitute an important process in a society, which we include in the limiting Fokker–Planck system. If the number of followers sharing an opinion exceeds a given threshold, leaders can emerge. If the density of ordinary people sharing a particular opinion falls below a certain threshold, the number of leaders promoting this opinion declines.

We illustrate the behaviour of our model for opinion formation with the example of elections in Carinthia, the southernmost state of Austria. Carinthia's landscape of political parties shows an interesting peculiarity. In 1999 the right-wing Freedom Party of Austria (FPÖ) became the strongest party in Carinthia; since then, it has made continual gains in elections for the state assembly (Landtagswahlen), receiving almost 45% of the votes in 2008. This outcome was strongly influenced by the popularity of the party leader, Jörg Haider, a controversial figure frequently criticized in Austria and abroad as populist, extreme-right, even antisemitic, yet strongly acclaimed by his followers. Haider, who practically led the FPÖ single-handedly, was able to unite the political spectrum from conservatives to extreme-right and establish a governing party whose success was founded less on political ideology than on the authority of one opinion leader—Haider himself.

Table 1 shows the results of the state elections in 2004 and 2009. We set the initial distribution of ordinary people to Gaussian distributions centered on the respective part of the left–right political spectrum, with the weights of the Gaussian distributions chosen in accordance with the results of the Landtagswahlen in 2004 (see Table 1). Here, $w = -0.75$ corresponds to the Greens (Grüne), $w = -0.25$ to the Social Democratic Party of Austria (SPÖ), $w = 0.25$ to the Austrian People's Party (ÖVP), and $w = 0.8$ to the FPÖ. We assume the presence of several opinion leaders in the system, associated with the different parties but with different weights representing their influence.

The behaviour of the solution is depicted in Figure 1. We observe that in the presence of the stronger ÖVP leader, people move from the SPÖ to the ÖVP, while people with extreme opinions accumulate around the strong leader at $w = 0.8$. A small group of people split from the initial density at $w = 0.8$ (initially attracted by the strong leader at $w = 0.5$) and form a new group at $w = 0.7$. This can be interpreted as the separation of two parties associated with the emergence of a new opinion leader. This behaviour is similar to real events in Carinthia: In April 2005 Haider formed a new party, the Alliance for the Future of Austria (BZÖ), with himself as leader, thereby de facto splitting the FPÖ into two parties. Haider died in a car crash in October 2008. In the elections of March 2009, the BZÖ, strongly invoking its deceased leader, managed to enlarge its share of votes to 44.9%, while the FPÖ failed to enter the Landtag.

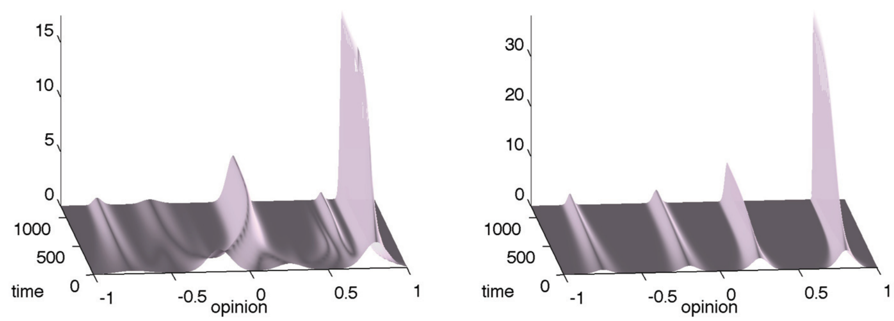


**Figure 1.** *Opinion formation as reflected in state elections in Carinthia, Austria. Left, evolution of ordinary people; right, evolution of opinion leaders.*

## References


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*B. Düring is a senior lecturer in mathematics at the University of Sussex, UK.*