

Something for Everyone in an Expert's Tour of the TSP

In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation. By William Cook, Princeton University Press, 2012, Princeton, New Jersey, 272 pages, \$27.95.

A salesman, given a list of cities he must visit, needs to calculate the shortest route that will take him through all the cities and back home. This is the traveling salesman problem. It is one of the easiest of all algorithmic problems to describe. Recently, my colleague Richard Cole and I needed to write a explanation, for a general audience, of the concepts of an intractable problem, an approximation algorithm, and limits on the best possible tractable approximation algorithm. Using the TSP as an example, we were able to explain all of them in a couple of paragraphs. It is hard to think of another problem that would have served nearly as well.

BOOK REVIEW

By Ernest Davis

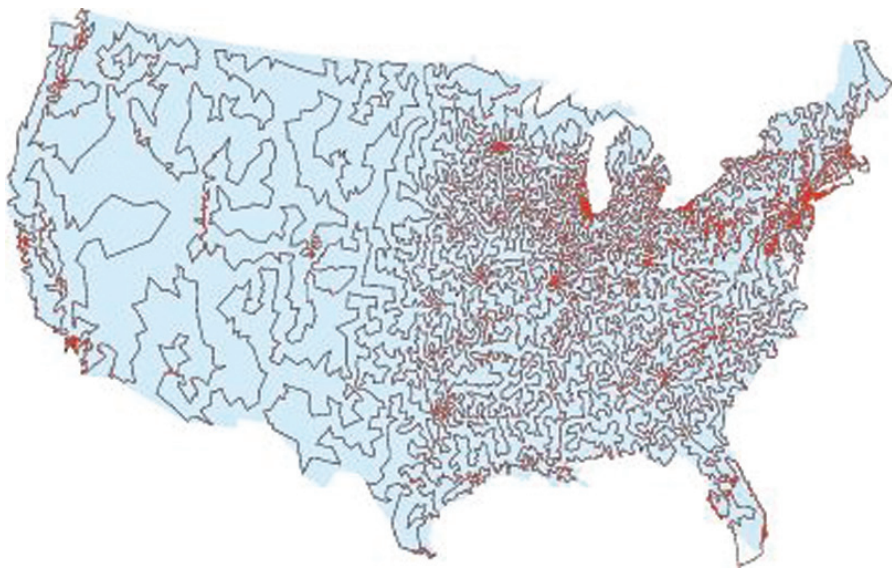
William Cook is a world expert on techniques for solving large instances of the TSP and one of the authors of the definitive work on the problem.* *In Pursuit of the Traveling Salesman* re-presents the material for a more popular audience, from many viewpoints: computational, mathematical, historical, aesthetic, and psychological.

The mathematical history of the TSP involves many great names in mathematics and computer science. Front and center is George Danzig (also the inventor of the simplex method), who in 1954, with Ray Fulkerson and Selmer Johnson, solved by hand an instance of the TSP passing through one city in each of the 48 contiguous states plus Washington, DC, and proved that their solution was optimal. Euler, Gauss, and Hamilton figure in the early history of the problem. More recently, Julia Robinson showed in 1949 how linear programming could be applied to the TSP; Richard Karp both proved, in 1971, that the TSP is NP-complete, and formulated, in 1962, what is still the best complete algorithm known (running time $O(n^{2^n})$); and Brian Kernighan and Shen Lin, in the early 1970s, entirely changed the landscape by developing heuristic techniques for finding efficiently near-optimal solutions to very large instances (tens or hundreds of thousands of cities). The current stars in the race to solve very large TSPs—the field's Roger Federer and Rafael Nadal—are Keld Helsgaun of Roskilde University, Denmark, and Yuichi Nagata, of the Japan Advanced Institute of Science and Technology.

Cook's explanation of the ideas used in solution techniques is wonderfully clear, and he has supplemented the text with excellent color figures. Any reader with a basic background in algorithm theory and elementary mathematics can work through these descriptions and get a clear idea of the issues and techniques involved. The book also explores many other aspects of the problem: numerous applications of the TSP, including DNA analysis, the printing of computer chips, and the planning of telescope motions; itineraries of actual 19th- and 20th-century salesmen and other travelers, including Abraham Lincoln's circuit as an Illinois lawyer; artists, such as Julian Lethbridge and Robert Bosch, who have used the TSP as a starting point for works of art; biochemical experiments in which DNA is used to solve instances of the TSP; and cognitive psychological experiments performed to test how well humans and animals do in solving the TSP in a natural setting.

The one aspect of the TSP that, rather strangely, is not considered in the book is variants of the problem. Cook sticks strictly to the metric version, in which the distances between cities satisfy the triangle inequality, or the stronger Euclidean version, in which cities are at specified points in the plane. He does mention in passing the problem of finding the shortest path that goes through all 48 states (viewed as extended regions, as distinguished from cities, idealized as points), but he does not at all discuss the solutions to this problem or other variants.†

Many parts of Cook's book are readable even for readers with little or no knowledge of computer science and mathematics; many parts will be new even to experts in the area. It is a fine example of how to write popular computer science and applied mathematics in a way that is both informative and entertaining.



A tour of all 13,509 U.S. cities having population of at least 500. From *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation*.

**The Traveling Salesman Problem: A Computational Study*, by D. Applegate, R.E. Bixby, V. Chvátal, and W.J. Cook, Princeton University Press, 2006.

†A fine survey of TSP variants can be found in *The Traveling Salesman Problem and its Variations*, G. Gutin and A.P. Punnen, eds., Springer, 2006.