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Obituaries: Olga Ladyzhenskaya

Olga Ladyzhenskaya during a recent visit to Iowa State University.

Olga Ladyzhenskaya, one of the great mathematicians of her or any generation, died on January 12, 2004, a few weeks short of her 82nd birthday. Ladyzhenskaya deeply influenced mathematics and mathematicians in every possible way. She advised many graduate students and junior researchers, most of whom have gone on to become important contributors in their own right. For many years she headed a group of outstanding researchers who were (and remain) at the forefront of research in partial differential equations and mathematical physics. She wrote several books [1–6] that to this day serve as primary sources for research in partial differential equations.

Most of all, however, Ladyzhenskaya exerted her influence through the large number of seminal papers she wrote on analyses of partial differential equations and on the development of functional analytic tools for performing those analyses. Through these papers, she played a primary role in changing the way partial differential equations are examined; for example, she was perhaps the leading figure in the popularization of the notion of weak solutions of partial differential equations.

Olga Alexandrovna Ladyzhenskaya was born on March 7, 1922, in Kologriv, a small Russian town. Her father, a mathematics teacher, helped form her life-long interest in the subject. In 1937, however, he was arrested by Stalinist authorities and executed without trial. (The incident is chronicled early in Solzhenitsyn's *The Gulag Archipelago* [7].) This tragedy deeply affected Ladyzhenskaya throughout her life. In 1939, she graduated with honors from Kologriv's secondary school but, being the daughter of a perceived enemy of the state, was refused admittance to Leningrad State University. Somehow, she did manage to enter the Leningrad Teachers' Institute, where she studied for two years before war-time conditions obliged her to return to Kologriv, where she taught mathematics in the secondary school.

In 1943, Ladyzhenskaya entered Moscow State University, from which she graduated in 1947. The scientific atmosphere she encountered there cemented her love of mathematics and proved ideal for the flowering of her talents. She began by studying algebra and number theory, but soon turned her attention to partial differential equations. She prepared her Diploma Thesis on a problem suggested to her by I.G. Petrovskii.

In 1947, for family reasons, she moved to Leningrad and became a graduate student at Leningrad State University. There, she began her longstanding collaboration and friendship with V.I. Smirnov. She was greatly influenced by the focus among Leningrad mathematicians on the study of the equations of mathematical physics. The subject of Ladyzhenskaya's doctoral dissertation, which was supervised by S.L. Sobolev and which she defended in 1949, was the development of finite difference methods for linear and quasilinear hyperbolic systems of partial differential equations. Afterward, she continued to study boundary-value and initial boundary-value problems for partial differential equations of all types. Several of the results she obtained were summarized in a monograph that she used as her Dr. Sc. thesis, which she defended in 1953 at Moscow State University. In 1951, she proved her famous second fundamental inequality for elliptic operators of order two with smooth coefficients, i.e.,

$$\|u\|_{W_2^2(\Omega)} \le C(\Omega)\left(\|\mathcal{L}u\|_{L^2(\Omega)} + \|u\|_{L^2(\Omega)}\right)$$

for arbitrary functions $u \in W_2^2(\Omega)$, satisfying one of the homogeneous classical conditions on the boundary of the domain Ω.

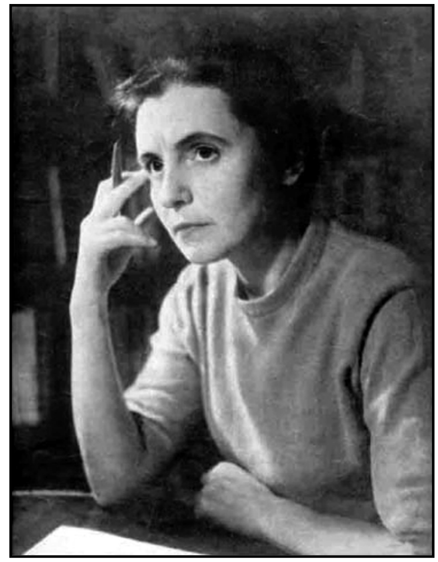
In 1954, Ladyzhenskaya began working at the Leningrad (later renamed the St. Petersburg) Branch of the Steklov Mathematical Institute, where, from 1961 to 1999, she headed the Laboratory on Mathematical Physics. In 1959, she was instrumental in the resumption of activities of the Leningrad (now the St. Petersburg) Mathematical Society. She served as the society's vice-president from 1959 to 1965 and from 1970 to 1990, and as its president from 1990 to 1998. For political reasons, Lady-zhenskaya was not allowed to travel outside the Soviet sphere of influence until 1958 (when she attended the International Congress of Mathematicians in London) and then not again until 1988, another 30 years!

Throughout the 1950s and 60s, Ladyzhenskaya continued to obtain important results about the existence and uniqueness of solutions of linear and quasilinear elliptic, parabolic, and hyperbolic partial differential equations. She also studied various systems that arise in mathematical physics, among them the equations of elasticity, the Schrödinger equation, the linearized Navier–Stokes system, and the Maxwell system.

At this point, Ladyzhenskaya turned her attention to the Navier–Stokes equations, which were to interest her for the rest of her life. In the steady-state case, she proved the global solvability of problems in bounded and exterior domains. For initial boundary-value problems, she proved the unique solvability for small enough time, as well as the global unique solvability for small enough data. In dimension two, with the help of the multiplicative inequality

$$\|u\|^2_{L^4(\mathbb{R}^2)} \le C\|u\|_{L^2(\mathbb{R}^2)}\|\nabla u\|_{L^2(\mathbb{R}^2)},$$
$$\forall u \in C_0^\infty(\mathbb{R}^2),$$

which now bears her name, she proved the global unique solvability. In these studies, she developed the rigorous theory for spaces of solenoidal vector-valued fields and proved several new a priori estimates. These and other results on the Navier–Stokes equations formed the basis of her celebrated monograph on incompressible viscous flows [2], undoubtedly one of the most important and influential mathematics books ever published. She constructed the first convergent finite difference schemes for the two-dimensional and three-dimensional Navier–Stokes equations and later analyzed a method of fractional steps.



Olga Ladyzhenskaya at Leningrad State University. Photograph from her personal archives.

Ladyzhenskaya believed that, in three dimensions and for large Reynolds numbers, the Navier–Stokes equations do not give a complete and correct description of the dynamics of fluids; one of the serious arguments supporting this belief was her own result on non-uniqueness in the class of weak Hopf solutions. This led her to propose modifications to the Navier–Stokes system, which she presented at the International Congress of Mathematicians in Moscow in 1966. For these modified Navier–Stokes equations, she proved the global unique solvability of the important boundary-value and initial boundary-value problems.

Her study of fluids problems led to much other work with her students on magneto-hydrodynamics, the stability of fluid flows, and the Navier–Stokes equations in domains with noncompact boundaries. In the 1970s, she turned her attention to attractors for the Navier–Stokes equations in two dimensions and for her modifications of the Navier–Stokes equations in three dimensions. On the whole, the methods she developed for the study of the Navier–Stokes and related equations proved to be applicable to many other dissipative problems.

From the mid-1950s on, Ladyzhenskaya, along with her students, made great advances in the study of boundary-value problems for quasilinear elliptic and parabolic equations. They developed a very complete theory for the solvability of boundary-value problems for uniformly parabolic and uniformly elliptic quasilinear second-order equations and of the smoothness of generalized solutions. A particular result was a solution of Hilbert's 19th problem for one second-order equation. It turned out that these methods are also effective in the study of certain classes of non-uniformly elliptic equations.

Ladyzhenskaya also wrote many papers in other areas of mathematics, among them perturbation problems, diffraction problems, the Galerkin method, the foundations of the principle of limiting amplitude, the one-dimensional equations of gas dynamics, the spectral model of Friedrichs, the Euler equations, a problem with a free surface over a sphere, collapses in nonlinear problems, global unique solvability of the Cauchy problems for one-dimensional systems of the theory of chiral fields, and the foundations of the Coleman principle for finding symmetric solutions of many-dimensional variation problems. In the 1980s and 90s, she continued her study of attractors for partial differential equations, and in particular for the Navier–Stokes equations in two dimensions and her modifications of the Navier–Stokes equations in three dimensions.

Remarkably, Ladyzhenskaya remained active until the very end of her life. In fact, she had formulated very ambitious research plans for the next five years, and it is a great loss to mathematics that she did not live long enough to carry out these plans.

Olga Ladyzhenskaya during a recent visit to Florida State University (with Matthew Aresco and Margaret Gunzburger of the Lake Jackson Ecopassage Alliance). Photo courtesy of Matthew Aresco.

Ladyzhenskaya was honored for her work on many occasions, both in the Soviet Union (and then in Russia) and abroad. In 1954 she was awarded the first Prize of the Leningrad State University; she received the same prize again in 1961, followed by the Chebyshev Prize of the USSR Academy of Sciences and the State Prize of the USSR in 1969 (the latter three jointly with N. Ural'tseva). In 1981, she was elected a corresponding member of the Academy of Sciences of the USSR. She was elected a foreign member of the Deutsche Academie der Naturforscher Leopoldina in 1985 and of the Academia Nazionale dei Lincei in 1989, a full member of the Russian Academy of Sciences in 1990, and a foreign member of the American Academy of Arts and Sciences in 2001. She received the S.V. Kovalevsky prize in 1992, an honorary doctorate from the University of Bonn in 2002, and the Golden Lomonosov Medal, the Ioffe Medal, and the St. Petersburg University Medal in 2003. In 1998, she delivered the John von Neumann Lecture at the SIAM Annual Meeting in Toronto.

Outside of mathematics, Olga had great interest in art, poetry, music, and literature. A very active member of the intellectual community in St. Petersburg, she counted among her close friends Aleksandr Solzhenitsyn and the noted poet Anna Akhmatova. She was a passionate lover of animals big and small, exotic and mundane. She took great joy in watching squirrels climb trees, feeding sea gulls out of her hand, and having personal encounters with alligators. (On one visit to the U.S., she expressed the desire to see a skunk in the wild; in Russia, she had seen them only in zoos. Fortunately, she was dissuaded from getting too close a look.)

Olga dominated any gathering she was a part of, not through bluster and noise, but by virtue of her genius, will, charm, and charisma. In her own personal behavior and character, she maintained the highest standards; this, along with her extraordinary abilities and accomplishments, helped her become a great and influential mathematician, despite the great tragedy and the large, politically motivated obstacles she often had to overcome. She was justifiably proud of her accomplishments, but she never let her pride affect her interactions with others; she was devoted to her family and friends, and countless mathematicians benefited from her patient, kind, and selfless help. This gentlewoman was a gentle woman whose extraordinary talents will be sorely missed by the mathematical community; her family, friends, colleagues, and, indeed, everyone who knew her will miss her much more as a person.

References

[1] O. Ladyzhenskaya, *The Mixed Problem for a Hyperbolic Equation*, Gostehteorizdat, Moscow, 1953.

[2] O. Ladyzhenskaya, *The Mathematical Theory of Viscous Incompressible Flow*, Fizmatgiz, 1961. English translation: Gordon and Breach, New York, 1963; 2nd Ed.: Gordon and Breach, New York, 1969.

[3] O. Ladyzhenskaya and N. Ural'tseva, *Linear and Quasilinear Elliptic Equations*, Nauka, Moscow, 1964. English translation: Academic Press, New York, 1968.

[4] O. Ladyzhenskaya, V. Solonnikov, and N. Ural'tseva, *Linear and Quasilinear Equations of Parabolic Type*, Nauka, Moscow, 1967. English translation: American Mathema-tical Society, Providence, Rhode Island, 1968.

[5] O. Ladyzhenskaya, *The Boundary Value Problems of Mathematical Physics*, Nauka, Moscow, 1973. English translation: Springer-Verlag, New York, 1985.

[6] O. Ladyzhenskaya, *Attractors for Semigroups and Evolution Equations*, Cambridge University Press, Cambridge, UK, 1991.

[7] A. Solzhenitsyn, *The Gulag Archipelago*, Harper & Row, New York, 1973.

—Max Gunzburger, Florida State University, and Gregory Seregin, Steklov Institute of the Russian Academy, St. Petersburg Branch,

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