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The Tripos in a Century of Mathematical Physics at Cambridge University

Masters of Theory. Cambridge and the Rise of Mathematical Physics. *By Andrew Warwick, University of Chicago Press, Chicago, 2003, 520 pages, $29.00.*

BOOK REVIEW

By Ivor Grattan-Guinness

A well-known feature of the history of British science, especially during the 19th century, is the dominating position of Cambridge University and within it of the Mathematics Tripos, the competitive graduation examination process that ranked candidates in order of "Wrangler" (and two lower orders) by the total number of marks gained. Senior Wranglership was star status for the year, though many of its holders made few or no contributions to research in later years. All the major Cambridge research figures did gain high Wrangler places, however. Some of them also won the Smith's Prize, an annual essay competition that, among other roles, strengthened still further the place of applied mathematics within the mathematics curriculum. The candidate at the bottom of the list was also a celebrity, parading around the Cambridge streets carrying his "wooden spoon" made out of an oar.

The Tripos has featured in biographies of several figures—Lord Kelvin, for example—and in histories of the development of various areas of mathematics and physics. The system itself has received much less attention, although some pioneering historical work was effected in the writings by W.W. Rouse Ball on Cambridge mathematics (1889 and later), and later in Peter Harman (ed.), *Wranglers and Physicists* (1985).

The book under review transforms this situation, with a treatment of mathematical physics in which the Tripos plays a major role, from the revival of mathematics and science at Cambridge in the early 19th century to the close of the Tripos system in 1909 and some of the research pursued thereafter. The manner of teaching and examining is covered in detail, as is the place of ex-Wranglers' Tripos training in their later conception and pursuit of research problems. The book treats the culture in some depth: These were truly manly mathematicians, many of whom also pursued sports seriously. The (non-)place of women is noted: Barred from receiving degrees in any subject until 1948, they were ranked "between" the nth and $(n + 1)$th Wranglers for some n—for $n = 0$ in the famous case of Philippa Fawcett in 1890.

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The many illustrations include not only portraits of some Wranglers (famous later or not) but also pertinent lecture notes, research publications, and Tripos examination questions and even candidates' attempts to answer them. This last, particularly welcome, source is one result of the author's extensive use of manuscript collections. The index and bibliography are well done, though the items for Albert Einstein are cited only by their numbers in the current edition of his works, so that the chronology is not too clear.

In the rest of this review I consider three aspects of the story, pointing to strengths of the book under review and also indicating a few matters that the author may have neglected somewhat. I end this section with three of the latter. Firstly, as the author stresses, the education system was based on paper, in both manuscript and published form; thus, more attention might have been given to the publishers of (text)books and journals. The University Press was fairly minor here, with commercial houses, such as Deighton's [5] and Macmillans, prominent. Secondly, another early career instrument was the dissertation written by the (usually) recent graduate to secure a fellowship at a college; these are rarely mentioned, though one reason is that many of them were not published.

Finally comes a cultural matter, the (non-)relationship of research with Christianity. Wranglers were usually Christians—until reforms beginning in the 1870s, adhesion to the Anglican brand was obligatory—and religion had once played a role in the development of mathematics. Yet from around 1750, the importance of religion in mathematics had decreased very considerably everywhere [3], although it *rose* in other sciences, such as geology and biology. The case of G.G. Stokes is typical: An aetherian mathematical physicist, fervent Christian, and the Gifford Lecturer on science and religion in the early 1890s, he asserted connections between his mathematics and his religion neither there nor elsewhere.

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The revival of mathematics in the early 19th century chiefly concerned the switch from Newton's "fluxional" version of the calculus to Continental versions. The change is not just to an algebraic formulation, as suggested by Warwick, however, for two different versions were adopted [1]. In the differential and integral version as conceived by Leibniz and modified especially by Euler in the mid-18th century, dx was an infinitesimally small variable increment on the variable x and $\int x$ an infinitely large variable formed by taking an infinite sum of xs, $d\int x = \int dx = x$, dy/dx was literally a quotient of differentials, and so on. While highly symbolic, the theory was construed within a geometrical framework: x was a line in space, y a force in space, and so on. Now this theory placed faith in the weird and wonderful in-finitesimals; finding this unacceptable, Lagrange from the 1770s advocated an alternative version that reduced the calculus to an algebra: The "derived functions" $f'(x)$, $f''(x)$, . . . were *defined* from the coefficients of the Taylor power-series expansion of which he believed that every function $f(x)$ was always capable, and integration was the reverse process ($f(x)$ was the integral of $f'(x)$, and so on).

Among the Cambridge reformers of the 1810s, John Herschel and Charles Babbage advocated Lagrange's version and worked to construct related theories; beginning in the 1840s, further developments became quite an English preoccupation, involving several Wranglers [4]. This was largely pure (or applicable) mathematics: It needs to be distinguished from the differential/integral version that applied mathematicians usually adopted, and not only at Cambridge after the reform, for its unrivalled flexibility in helping in the formation of scientific theories.

Both versions of the calculus were taken up at Cambridge in the forms then current in France, which was by far the dominant mathematical country in the world [2]. The author acknowledges this status in places, but rather in passing; however, it is substantial enough to need a specific summary, for the French dominance covered most areas of pure and applied mathematics of the time—in particular, the formation of much of mathematical physics outside mechanics. An interesting irony in the calculus is also evident: Exactly when Cambridge was adopting two French versions of the calculus, Cauchy in Paris was proposing a fourth version, like Newton's in its reliance upon limits (though cast in a far more refined form than Newton's own), that gradually gained friends in Britain.

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The single main novelty of the book lies in its account of the training for the Tripos examinations (and staff and students did think of it as training), not only the content of the curriculum but also the ways in which the coaches drilled the students. It was not obligatory for a student to employ a coach, but the vast majority seem to have done so. The role of coaches is finely captured, not only in the text, but also in an appendix that lists the first ten Wranglers for each year from 1865 to 1909, together with their coaches and colleges (with students from Saint John's and Trinity dominating). Coaches played a far greater role in the system than non-coaching college Fellows, whose own places in Cambridge mathematical life remain rather obscure.

Mechanics was an important part of the curriculum. Newton's version, despite its relaunching by William Whewell against French competitors in the late 1810s, was not necessarily adopted at Cambridge. In the 1840s Whewell also advocated inclusion of engineering mechanics in the curriculum, which is largely ignored in this book.

The most successful coach, as reckoned by the number of high Wranglers trained, was E.J. Routh, whose reign extended from the mid-1850s to the mid-1880s. During term time, he worked literally from morning to night. Against his opinion, the system from 1910 onward was changed to a ranking of candidates into three classes, and by alphabetical order within each class. His textbooks on mechanics remained as important reference works, and not only at Cambridge.

While the emphasis always lay upon applied mathematics, the curriculum gave a significant place to pure and applicable mathematics, especially the calculus in some form and its use in differential equations. Perhaps a little more could have been indicated about this category. In a nice example, the final list of Wranglers (1909) is reproduced from *The Times*. The author's caption emphasises one of the joint fourth Wranglers, a son of the applied mathematician G.H. Darwin (himself a son of Charles); nothing is said about Wranglers 1–3, who later pursued more distinguished careers, but mainly in pure mathematics.

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A major theme in mathematical physics of the 19th century, including in due course the Tripos curriculum, is electromagnetism and, at Cambridge especially, the impact of the work of Wrangler J.C. Maxwell from the 1860s onward. The rest of the book is dominated by consequences of this work, up to the adoption and adaptation of relativity theory during the 1920s (the end of the period studied is not quite clear).

Hitherto, much of the historical work on relativity has concerned Einstein and his German-speaking background and context; here the Cambridge side is treated. In particular, the author discusses J.J. Larmor, who had his own view of aether contraction; and when relativity theory was taken up at Cambridge, mathematicians adopted a formulation rather different from Einstein's, as an adjunct to an electronic theory of matter. Especially welcome are the accounts of the contributions of E. Cunningham and J.W. Nicholson, which came to be rather eclipsed by the ideas of A.S. Eddington, whose reading depended more upon Einstein's assumptions and differential geometry. Little or nothing is said about A.N. Whitehead, whose career in physics started in the mid-1880s with a (now lost) fellowship dissertation at Trinity on Maxwell's theory; around 1920, Whitehead developed his own version of relativity theory, which mapped it onto Euclidean space.

A casualty of the design of the latter part of this book is that much of the applied mathematics not linked to electromagnetism and relativity theory is sidelined. Thus, little or nothing is said about the practitioners of other parts of Cambridge mathematical physics, such as continuum and celestial mechanics, from the 1870s to the 1920s. The topics and figures involved include E.T. Whittaker, who in 1903 co-launched an important series, "Cambridge Tracts in Mathematics and Mathematical Physics," published by the Cambridge University Press; the effect of the Great War in stimulating projectile theory among such mathematicians as H.W. Richmond and E.A. Milne; and the early work of H. Jeffreys and G.I. Taylor. One looks forward to a general history of post-Wrangler mathematical physics at Cambridge.

References

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