From *SIAM News*, Volume 37, Number 7, September 2004

Imaging Science

Morphological Image Registration and Nonlinear Elasticity

By Marc Droske and Martin Rumpf

Classic modes of image acquisition like computer tomography and magnetic resonance tomography, along with a variety of novel image sources, such as functional magnetic resonance imaging, three-dimensional ultrasound, and densimetric computer tomography, provide a variety of three-dimensional, usually scalar, grey-valued images of the human body. Even when such images show the same objects, however, corresponding parts of the objects typically do not appear in the same position.

For many reasons, beyond differences in body positioning in the image-acquisition devices, we cannot simply overlay images. Physical deformation of the object structures, temporal changes in the object from one acquisition time to the next, and unknown distortions due to artifacts in the image generation itself usually make it impossible to overlay images, even after a suitable rigid body motion. Hence, we need a nonlinear transformation that ensures the correlation of physically related object structures in the images. Such correlation becomes particularly challenging when the images under consideration are of physically different objects with similar structures, such as images of different human brains. We cannot usually expect a correspondence of image intensity values at structurally corresponding positions (compare the CT and MR slices in Figure 1). What remains, at least partially, in real applications is the local image structure, or "morphology," of corresponding objects. Our aim is thus to match images with respect to their morphologies.

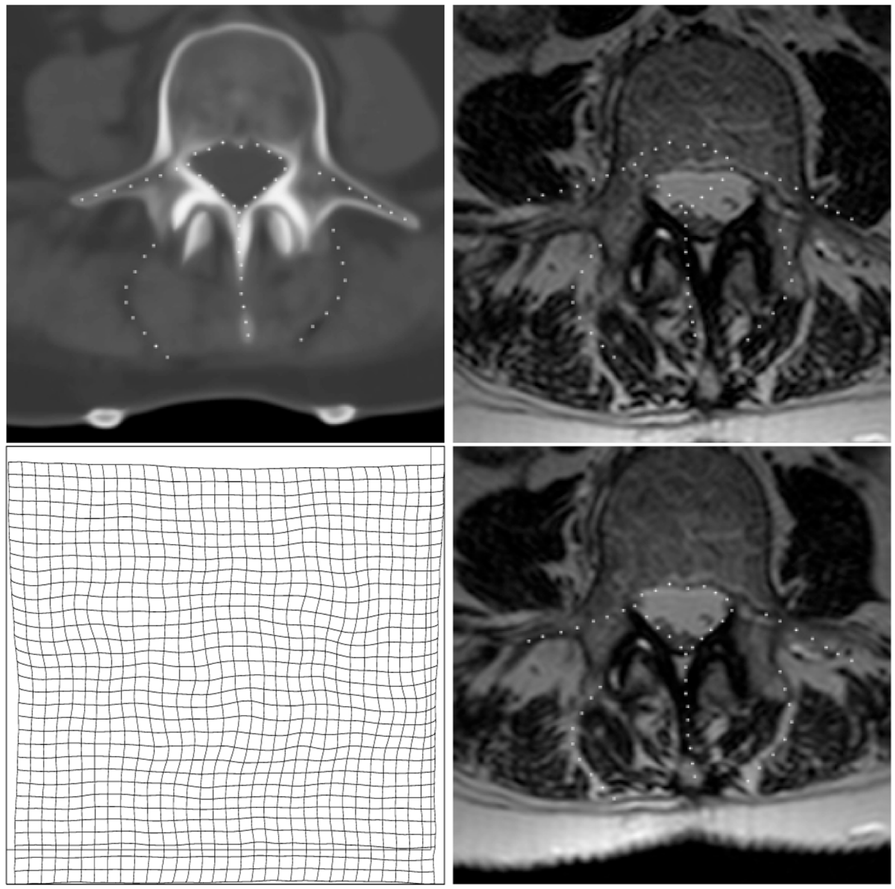

Figure 1. *Sectional morphological registration of a pair of MR and CT images of a human vertebra. Top left: reference, CT. Top right: template, MR. Bottom left: resulting deformation applied to a lower-resolution uniform grid. Bottom right: result of the registration. The dotted lines indicate structure originally found in the CT reference image; they are always drawn at the same position and indicate the proper matching.*

A Variational Formulation

Mathematically, we take a variational approach to this problem. Our aim is to correlate two images—a reference image R and a template image T, for simplicity defined here over the same image domain Ω.

In fact, we seek a deformation ϕ from the reference domain onto the template domain, such that

$$\text{Morphology}(T \circ \phi) \approx \text{Morphology}(R).$$

Before we can state the problem of morphological image matching in the language of the calculus of variations, we need to define the morphology of an image mathematically. To this end, we consider level sets M_c of an image I on an image domain Ω corresponding to an image intensity c, given by

$$M_c[I] := \{x \in \Omega \,|\, I(x) = c\}.$$

We can now define an image I and an image R as morphologically equivalent if there exists a (usually monotone) grey-value transformation β such that $\beta \circ I = R$.

Such a grey-value transform does not change the shape of the level sets; it simply exchanges level sets of different intensity values ($M_c[R] = M_{\beta(c)}[I]$). Hence, a morphology $M[I]$—which identifies the structure of an image but remains invariant under grey-value transforms—can be identified with the set of all level sets

$$\text{Morphology}[I] := \{M_c[I] \mid c \in \mathbb{R}\}.$$

The morphology $M[I]$ can be identified with the normal field (Gaussian map) $n_I : \Omega \to \mathbb{R}^d; x \mapsto \nabla I / \|\nabla I\|$ up to the orientations of the normals. Indeed, the normal field uniquely identifies the set of level sets. A rigorous treatment of possible singularities of the normal field can be found in [4].

Because morphological methods in image processing are characterized by invariance with respect to morphology, geometry naturally enters the field of morphological image processing [6]. Aiming for a morphological registration method, we look for a deformation $\phi : \Omega \to \Omega$ such that each deformed level set $\phi(M_c[R])$ of the reference image R coincides with a level set $M_{\tilde{c}}[T]$ of the template image. In terms of the normal field, we can restate this as a requirement that the deformed normal of the reference image n_R^ϕ be aligned with the normal of the template image $n_T \circ \phi$ at the deformed position; the geometric configuration is sketched in Figure 2. Normals are transformed by the co-factor $\text{Cof}\, D\phi = \det D\phi\, D\phi^{-T}$, and we get $n_R^\phi = \text{Cof}\, D\phi n_R / \|\text{Cof}\, D\phi n_R\|$. Hence, a suitable first choice for a morphological matching energy is given by the least-square difference

$$E_m[\phi] := \int_\Omega \left\| n_T \circ \phi - n_R^\phi \right\|^2 dx.$$

We are not restricted to this type of energy, however, and can take non-monotone grey-value transforms into account in our notion of morphology.

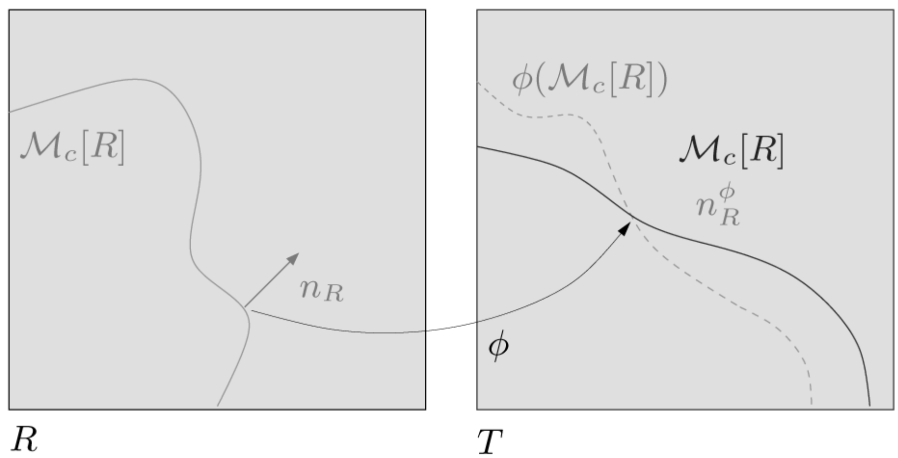

Figure 2. *The geometric configuration in morphological image matching and the mismatch of deformed reference normal and template normal at the deformed position.*

Regularization

Any deformation that merely exchanges level sets, or that only reparam-etrizes level sets, does not change the energy. Thus, given one minimizer we already have many, and the structure of the set of minimizers is very irregular. Seeking a regularization of the energy that will overcome this problem, we consider a second energy E_{reg} and look for minimizers of the combined energy $E_m + E_{reg}$.

A suitable class of regularization energy consists of polyconvex energy functionals, which are well known from the theory of nonlinear elasticity [2]. As an example, we consider an energy controlling the change in length, area, and volume separately:

$$E_{reg}[\phi] := \int_\Omega \underbrace{a\|D\phi\|_2^p}_{(\text{length control})} + \underbrace{b\|\text{Cof}\, D\phi\|_2^q}_{(\text{area control})} + \underbrace{\Gamma\left(\det D\phi\right)}_{(\text{volume control})} dx.$$

In particular, with $\Gamma(D) = D^r + D^{-s}$ for $r, s > 0$, we can penalize extensive volume shrinkage and expansion simultane-ously. The outstanding property of regularization energy of this type is that it prevents the interpenetration of matter and thus enables us to ensure that minimizers of the energy are homeomorphisms [1]. It also allows us to control singularities in the normal field, such as edges or boundaries of flat regions. Furthermore, the term responsible for the area element transformation is again the cofactor matrix of the Jacobian. We can thus treat the minimization problem via direct methods in the calculus of variations, even though the regularization is actually not of higher order, as usually observed in energy regularization approaches.

The numerical implementation is based on a multiscale approach combined with a stepsize-controlled gradient descent method (see the registration results in Figure 1). In fact, we begin by matching coarse representations of the image data and then proceed with successively finer resolution [3]. Resolution of the coarse-scale representations of the images on coarse meshes turns out to be sufficient, which significantly reduces the computation time.

Relation to Information Theoretic Registration Models

Information theoretic approaches to multimodal registration have become very popular during the last decade. Viola and Wells [7], and Collignon et al. [5] independently, came up with the idea of interpreting the image intensities as random variables and then using the concept of mutual information to measure how well the deformed reference image describes the template image in a stochastic sense. For an infinitesimal volume, where the image can be approximated by an affine function, mutual information is maximal if the tangent directions coincide. Thus, in a multiscale treatment of the problem, we see strong links between the morphological approach and the mutual information approach that are especially worthy of further exploration.

Looking to the future, the real challenge is to generalize mathematical concepts of morphology, in such a way that they reflect the physical reality and the actual output of image-acquisition devices. In medical imaging especially, morphology is a framework that is not limited to mathematical morphology. Furthermore, multiscale methods, which truly reflect morphological structure, will be a key to improved efficiency and robustness.

References

[1] J. Ball, *Global invertibility of Sobolev functions and the interpenetration of matter*, Proc. Roy. Soc. Edinburgh, 88A (1988), 315–328.

[2] P.G. Ciarlet, *Three-Dimensional Elasticity*, Elsevier, New York, 1988.

[3] M. Droske, M. Rumpf, and C. Schaller, *Non-rigid morphological registration and its practical issues,* in Proceedings of ICIP ’03, IEEE International Conference on Image Processing, September 2003, Barcelona, Spain, II (2003), 699–702.

[4] M. Droske and M. Rumpf, *A variational approach to non-rigid morphological registration*, SIAM J. Appl. Math., 64 (2004), 668–687.

[5] F. Maes, A. Collignon, D. Vandermeulen, G. Marchal, and P. Suetens, *Multi-modal volume registration by maximization of mutual information*, IEEE Trans. Medical Imaging, 16:7 (1997), 187–198.

[6] G. Sapiro, *Geometric Partial Differential Equations and Image Analysis*, Cambridge University Press, Cambridge, UK, 2001.

[7] P. Viola and W. Wells, *Alignment by maximization of mutual information*, Int. J. Comp. Vision, 24:2 (1997), 137–154.

Martin Rumpf is a faculty member in mathematics at the University Duisburg-Essen in Germany. Marc Droske is a PhD student in his group.