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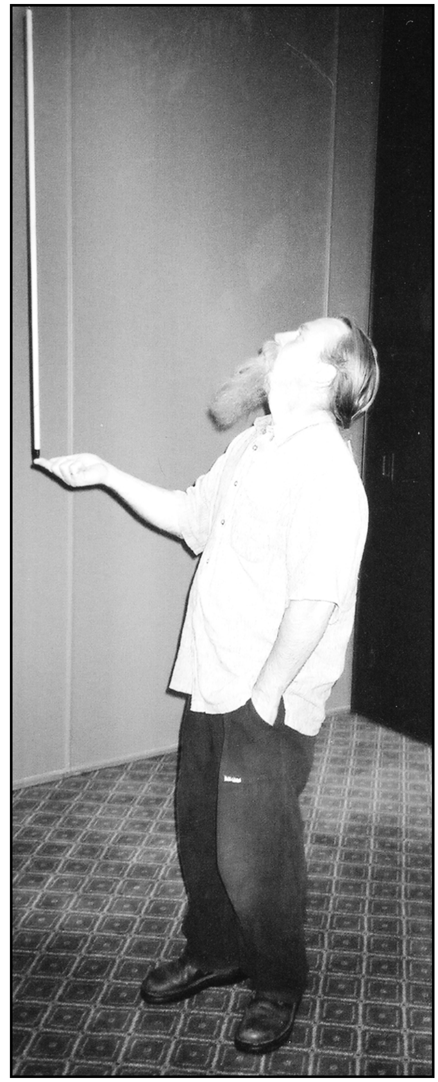
Life in the Balance

By Barry A. Cipra



Theoretical physicist Juan Luis Cabrera, an expert in the effects of noise in time-delay systems, demonstrated his stick-balancing expertise in Montreal.

Juan Luis Cabrera knows on–off intermittency when he sees it. So when John Milton, a neurologist at the University of Chicago Hospitals, showed him data from a series of stick-balancing experiments, the theoretical physicist, who had just joined Milton's group as a postdoc, spotted the effect immediately—or, more precisely, with just a short delay.

Milton and Cabrera, who is now at the Instituto Venezolano de Investigationes Científicas in Venezuela, described results of their collaboration in a minisymposium on delay differential equations at last year's SIAM Annual Meeting, held in Montreal in June 2003. Their research sheds new light on what happens in the nervous system when a person acquires new skills. The insights could eventually lead to clinical applications for victims of brain injuries who need to relearn certain motor skills. (In one of his more enjoyable roles, Milton runs a golf clinic for people who have had strokes.)

The experiments Milton's group carried out were very simple: They had volunteers (mainly students) practice balancing a 2-foot stick on their fingertips, and recorded their movements with two motion-capture cameras. The cameras detected infrared light bouncing off reflective markers at each end of the stick, so that a computer could triangulate the positions to get a fully three-dimensional picture. The resulting time series can be expressed in terms of the crucial dynamical variable: the angle the stick makes with respect to the vertical.

Cabrera, an expert in the effects of noise in time-delay systems, noticed burstiness in the time series and suggested on–off intermittency as a possibility. An effect discovered by physicists in the 1980s, on–off intermittency occurs, roughly speaking, when a control parameter is randomly or chaotically forced back and forth across a stability boundary. It is characterized chiefly by a special scaling property: In a log–log plot of the probability distribution for time intervals between successive crossings of an arbitrary threshold, the data will, in theory, fall along a line of slope –3/2.

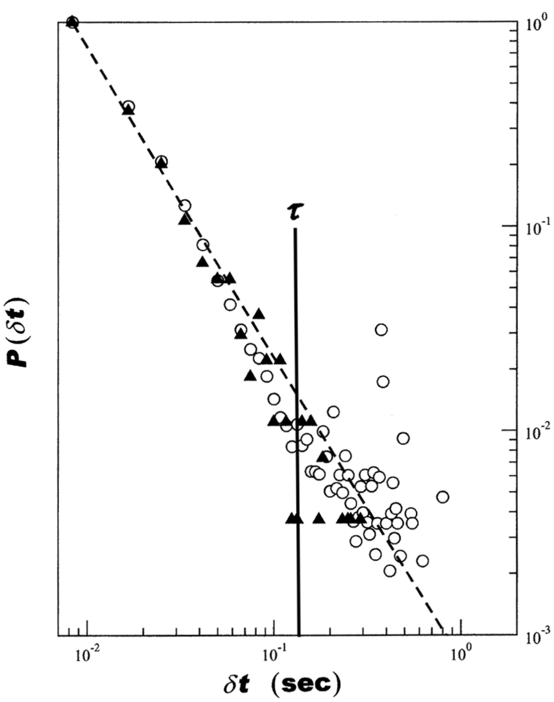


Figure 1. *Stick-balancing data agree with predictions of on–off intermittency. Approximately 98% of all corrective movements occur on time scales shorter than the delay time (vertical line).*

The researchers plotted their data and found that it lined up (see Figure 1). The questions then became, why does this happen and what does it mean?

For insight into the former question, Milton and Cabrera analyzed a mathematical model of stick balancing. The model is an inverted pendulum with noisy, time-delayed feedback. The role of time-delayed feedback in stick balancing is illustrated by the observation that longer sticks are easier to balance than shorter ones: Once the stick becomes sufficiently long, its rate of movement becomes slow relative to the time required by the nervous system to make corrective movements. When normalized by the delay time τ, the equation for the angle the stick makes with the vertical is $\theta'' + a\tau\theta' - b\tau^2 \sin\theta + R(t)\theta(t - 1) = 0$, where the coefficients a and b take friction and gravity into account and $R(t) = R_0 + \eta(t)$ represents the restoring force, which modulates an adjustable parameter R_0 with Gaussian white noise $\eta(t)$.

In the absence of noise, there is a region of stability in the (R_0,τ) plane lying roughly between a straight line and a parabola. When noise is added to a value of R_0 near the boundary, plots of the stochastic solutions resemble the experimental observations. In particular, the threshold crossings show the characteristic –3/2 scaling.

The significance of on–off intermittency in stick balancing lies in the implication that more than 98% of the "corrective" motions occur on time scales shorter than the delay time, which, based on cross correlations between the movements of the two ends of the stick, is estimated to be about 100 milliseconds. In effect, the nervous system exploits its own noise to enlarge the region of stability beyond its deterministic boundary. "What it's really doing is creating a little random walk on that boundary," Milton says. "It's a clever way of flipping the dice so you don't get too far away from the vertical position."

One apparent benefit of living on the edge is maneuverability. A perfectly balanced stick goes nowhere, while a stick that's kept slightly off-balance can be moved around. (Fighter pilots are well aware of the trade-offs between stability and maneuverability.) But Milton and Cabrera have found another, possibly greater benefit: With practice, stick balancers are able to *increase* their delay time—in other words, an experienced stick balancer pays less attention to the stick than a beginner does.

"Usually when people become highly skilled, their reaction times decrease," Milton says. "So we're going the wrong way!"

Milton conjectures that as the brain becomes accustomed to a task, it begins to economize. "We have just a certain amount of attentional resource, and we want to minimize the amount we can use for this task so we can do something else," he explains. In other words, "we're always trying to free up time."

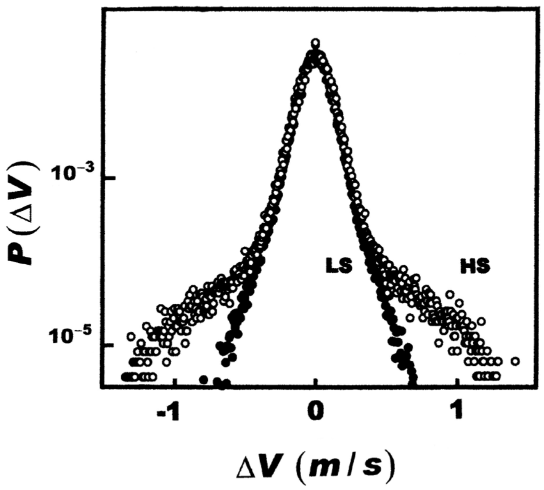


Figure 2. *Highly skilled (HS) stick balancers are able to make larger movements than lower-skilled (LS) novices.*

The stick balancer's hand motions reflect the increase in delay time. "The way the hand is moving is different between a low-skilled person and a high-skilled person," Milton says. "In particular, a high-skilled person begins to make large corrections."

Milton and Cabrera have quantified this observation in their studies. They have found that the (essentially two-dimensional) motion of the stick balancer's hand follows a process known as "Levy flight," a kind of random walk in which the probability of taking a step of length s is inversely proportional to a power of s. At both skill levels, the Levy flight is truncated—that is, large motions are suppressed (there are, after all, limits to how quickly a person's hand can move). But the truncation is significantly less for highly skilled individuals (see Figure 2)—having learned to move fast when they have to, experienced stick balancers don't need to react immediately to the motions of the stick.

Milton thinks the stick-balancing research could have implications for robotics—balance is a notorious problem for two-legged walkers—and even for the earthquake-proofing of buildings. (Sophisticated "shock absorbers," built into modern skyscrapers to counteract earthquakes, could conceivably be more effective with an intentional introduction of noise.) But the chief significance of the experiments, for now, remains in neurobiology, where there are far more questions than answers. Considering one of the most fundamental, Milton wonders, "What is the control problem that the nervous system is trying to solve?"

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