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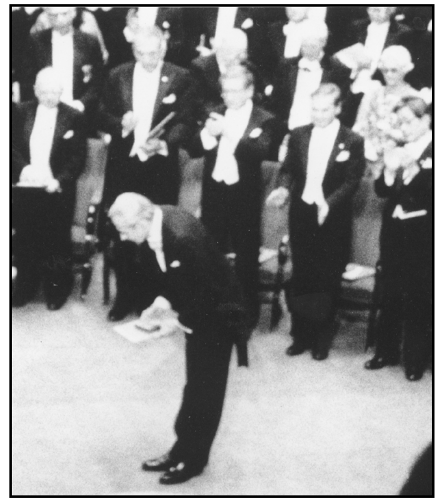
Evidence of a Rare Imagination

The Essential John Nash. *Edited by Harold W. Kuhn and Sylvia Nasar, Princeton University Press, Princeton, New Jersey, 2002, 244 + xxv pages, $29.95.*

Any mathematician who read *A Beautiful Mind*, Sylvia Nasar's acclaimed biography of John Nash, had to be looking for the appendices—the ones explaining what Nash actually did to earn his formidable reputation within the mathematical community. Well, here they are, in a beautifully produced volume from the Princeton University Press. The last three chapters (10–12) reproduce Nash's papers on real algebraic manifolds (1952), the imbedding problem for Riemannian manifolds (1956), and the continuity of solutions of parabolic and elliptic equations (1958). This is the work that, according to Nasar's biography, made Nash a strong contender for the 1958 Fields Medal. The biography described the conditions under which these works were produced, and Nash's interactions with the more experienced mathematicians who volunteered their services as mentors and sounding boards while he was producing them. The new volume presents the finished products, composed in attractive, easy-to-read type on archivable acid-free paper.



John Nash, Stockholm, 1994. The youthful work in game theory for which Nash received the Nobel Prize in economics is reproduced in the book under review, as is some of the later work believed to have made him a strong contender for a 1958 Fields Medal.

Another three chapters (5–7) reproduce the game theory papers for which Nash shared (in 1994, with John Harsanyi and Reinhard Selten) the Prize in Economic Sciences in Memory of Alfred Nobel. Kuhn has provided a brief introduction to these three chapters. "Equilibrium Points in *n*-person Games (chapter 5) is a two-page proof of the existence of what are now called "Nash equilibrium points" using Kakutani's fixed point theorem. It appeared in the *Proceedings of the National Academy of Sciences* for 1950. Nash's PhD thesis, "Non-Cooperative Games," was completed in the same year and is reproduced twice, first (chapter 6) in photostatic form, complete with hand-inserted symbols, and then (chapter 7) as published a year later in the *Annals of Mathematics*.

Two more chapters on game theory (4 and 8) concern two-player cooperative games. Chapter 4, "The Bargaining Problem," was Nash's first published work. It appeared in *Econometrica* in 1950, while he was still a graduate student, and seems to have grown out of a course in international trade that he took during his last term as an undergraduate at Carnegie Tech (now Carnegie Mellon University) to fulfill a distribution requirement. It was the only formal instruction in economics he ever received. Whether the project began as a term paper for the course or after his arrival in Princeton is now in doubt, according to Kuhn's introduction to the chapter. In any case, Nash showed the work soon after his arrival in Princeton to Morgenstern, who showed it to von Neumann, who agreed that it should be polished and published. The second paper on two-person cooperative games (chapter 8) develops the notion of "threat strategies."

Chapter 9, "Parallel Control," reproduces a RAND report dated August 27, 1954, during Nash's last summer as a consultant there. It contains what may be the first mention in print of parallel computing. Nash observes that, in order to compute a 100-dimensional inner product $a(1)b(1) + ... + a(100)b(100)$ in the usual sequential manner, it is necessary to perform 100 multiplications and 99 additions, for a total of 199 flops. If, on the other hand, one had access to 100 processors, one could perform the required 100 multiplications simultaneously, as if in a single step, and then carry out the 99 additions in only seven more simultaneous time steps, for a total of only eight "layers" of simultaneous computation. It is, by any measure, a substantial reduction in computing time. The report contains Nash's proposal for the architecture of such a parallel computer. Kuhn remarks, in his introduction to the chapter, that whether or not RAND's advanced computing group ever took Nash's suggestion seriously, the concept stands as evidence of a rare imagination.

The foregoing technical chapters occupy more than three quarters of the book. Chapter 3, also somewhat technical, was written by John Milnor. It concerns the game of Hex, which was invented by Piet Hein in Denmark and (independently) by Nash while at Princeton. There it was known as "Nash" and played with Go pieces on the hexagonally tiled washroom floors of Fine Hall, where a Hex board—a rhombus tiled by n^2 hexagons—of almost any size could easily be laid out. According to Milnor, $n = 14$ makes the most enjoyable game. Each player (White and Black; White to move first and alternately thereafter) strives to divide the board into disjoint parts by placing an unbroken chain of own-color tokens from his own to the opposite side of the board. Obviously, there can be no ties. Nash proved, in what Milnor describes as a "marvelously non-constructive manner," that, on an $n \times n$ board, White can always win. Also, on an $n \times m$ board, the player with the shorter distance to cover can always win.

The book's first chapter reproduces the announcement, dated October 11, 1994, that the Nobel prize in economic sciences for that year would be awarded to Harsanyi, Nash, and Selten. It concludes that "through their contributions to equilibrium analysis in non-cooperative game theory, the three laureates constitute a natural combination: Nash provided the foundations for the analysis, while Selten developed it with respect to dynamics, and Harsanyi with respect to incomplete information."

The second chapter begins with a brief autobiography and is followed by an amusing "photo essay," replete with pictures of Nash at various stages of his career, along with others who influenced his career, including Einstein, von Neumann, Morgenstern, Albert Tucker, Norbert Wiener, Harold Kuhn, David Gale, Jürgen Moser, and Ennio de Giorgi, as well as Nash's wife, Alicia, his two sons, and the actor Russell Crowe.

The book includes, in addition to its 12 chapters, a preface by Kuhn, an introduction by Nasar, and an afterword by Nash himself, along with appropriate back matter. Kuhn, Nasar, and the other contributors have performed a most welcome service by collaborating to bring together the pieces missing from *A Beautiful Mind*—pieces that Nasar alone could never have supplied. The mathematical community is eternally in their debt.—*James Case*