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The Roots of Computer Science in Symbolic Logic

Conversations with a Mathematician: Math, Art, Science, and the Limits of Reason. *By Gregory J. Chaitin, Springer-Verlag London Ltd., 2002, 158 pages, $29.95.*
Engines of Logic: Mathematicians and the Origin of the Computer. *By Martin Davis, W.W. Norton, New York, 2001 (paperback), 257+xii pages, $14.95.*

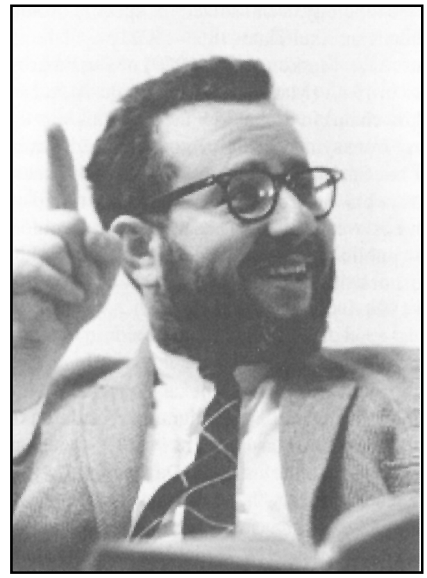
BOOK REVIEW
By James Case

The latest in Gregory Chaitin's series (four and counting) of nontechnical accounts of his work consists of 12 short chapters, of which three are (edited) lecture transcripts, three are TV interviews, and three are ordinary interviews. It also includes a short introduction, a few final thoughts, and some recommended further reading. Intended to be the most accessible in the series—all published by Springer—the new book is at times oracular (as in the chapter titled "Undecidability & Randomness in Pure Mathematics") and at others deeply personal.

In chapter 10, for example, when asked what mathematics is for him, Chaitin describes it as "fun"—especially when you've got an idea that seems to be leading somewhere—and "sensual," like music, albeit a form of music that most people are unable to hear. On the other hand, he observes, mathematics can resemble an adventure, like trying to climb a previously unscaled mountain. Sometimes you reach the summit, "where no one has been before, where the air is very pure, where the sky is a beautiful dark blue, where you get those incredible views, where you think you're closer to God, where you *understand*!" Yet things seldom turn out that way. More often than not, mathematical mountain climbers are frustrated by unforseen features of the landscape, which block the more obvious routes to the summit. This is a natural analogy for Chaitin, who climbs mountains himself "on weekends and holidays, especially in winter with snowshoes and ski poles."

Chaitin goes on to reflect that a career in mathematics can easily degrade one's performance as a husband, a father, and a member of the community:

> "Having mathematical talent is wonderful, but it's also a bit of a curse. It tends to take over your life. Now I didn't want it to take over my life. I think I tried to have a normal human life. But it did sort of take over my life, I guess."



Martin Davis, one of the "Hilbert solvers"(see review in this issue), is also the author of a recent book about the mathematical concepts underlying modern computers, reviewed here by James Case. Photograph from The Honors Class.

Perhaps to console himself for opportunities missed, he recalls the answer given late in life by Einstein—a confessed failure as husband and father—to the wife of a former colleague who wrote to ask why her beloved (and reputedly talented) spouse had "never accomplished anything." "But of course," Einstein replied, "it's because he's a good man."

Chaitin describes his thoughts on randomness, and some of the reactions to them. While studying physics as a teenager, he began to wonder what it meant for something to exhibit no pattern, to obey no law. His answer, when he found it, seemed entirely natural as applied to physics, where randomness is known and expected. It seemed somewhat unnatural, on the other hand, as applied to the foundations of mathematics, where randomness was not previously known or expected to occur. As a result, he says, he has become *persona non grata* in a portion of the logic community, where there is resistance to the conclusion that "God threw dice" even while constructing the natural numbers.

According to Chaitin, his most fruitful idea occurred to him in 1970, in Rio de Janeiro, just before Carnival time. During the Carnival itself, he hastens to inform his (Brazilian) interviewer, he was far too busy "dancing in the street and looking at those luscious carioca women" to think about mathematics. The idea was that, although almost every natural number does in fact satisfy his personal definition of randomness—much as almost every real number is transcendental—mathematical reasoning can prove only in isolated cases that this is so. He had discovered new limits, akin to those imposed by the incompleteness theorems of Gödel and Turing, on what mathematical reasoning can achieve!

The book's most controversial thesis, admittedly an exaggeration, is that "the computer was invented to clarify a question about the foundations of mathematics." He is referring, of course, to Alan Turing's invention of what he originally called "a-machines"—the "a" standing for "automatic." It is easily forgotten that Turing invented the machines that now bear his name to investigate certain fundamental issues raised by logicians (particularly Hilbert) at the beginning of the 20th century. Although Hilbert's attempt to encompass all of mathematics within a single formalization foundered on the reef of Gödel's incompleteness theorem, Chaitin asserts that his formalizations live on in the languages by which we communicate with modern electronic computers, forming an indispensable part of the century's most spectacularly successful technology. He describes these events as a largely forgotten chapter in intellectual history, and devotes his opening lecture to a summary thereof. After making more or less the same claim, Martin Davis devotes an entire book to the thesis.

Davis's book, which was originally published in 2000 under the title *The Universal Computer: The Road from Leibniz to Turing*, was recently reissued in paperback. It begins at the beginning, with a summary of Leibniz's long and varied career. In 1672, at the age of 26, Leibniz journeyed to Paris as the emissary of his patron, Baron Johann von Boineburg, nephew of the (vastly more influential) elector of Mainz. The purpose was to persuade Louis XIV to invade Egypt, rather than Germany, as Napoleon (disastrously) did a century later. Although the Baron soon died of a stroke, Leibniz contrived to remain in Paris for another four (exceedingly productive) years, during which he made two brief visits to London. On the first of the visits, according to Davis,

> "he was unanimously elected to the Royal Society of London based on a model he was able to exhibit of a calculating machine capable of carrying out the four basic operations of arithmetic. Although Pascal had designed a machine that could add and subtract, Leibniz' was the first that could multiply and divide as well. This machine incorporated an ingenious gadget that became known as a 'Leibniz wheel,' a device common in calculating machines well into the twentieth century.

In 1674, he described a machine that could solve algebraic equations, and he subsequently compared logical reasoning to a mechanical process. He stopped short, however, of describing an actual machine for performing deductions.

Christiaan Huygens, who was living in Paris at the time, supplied the youthful Leibniz with a reading list intended to acquaint him with the latest developments in mathematics. By the time he left France, in search of a new patron, Leibniz had discovered the rudiments of differential and integral calculus, along with the "fundamental theorem" relating them. In the process, he invented the notation (involving the symbols d and $\int$) that—because it is simpler and more suggestive than Newton's—students learn to this day. Davis makes much of the latter contribution, in part because he shares Leibniz's wonder at the manner in which appropriately chosen symbols "seem to magically produce correct answers to problems almost unaided." Later in life, Leibniz returned to these questions, resolving to invent a *calculus ratiocinator*, or algebra of logic. Indeed, in the last years before his death (in 1716), Leibniz made significant strides toward that goal. It is unfortunate that his search for a new patron led him to the Dukes of Hanover, who thought his time best spent writing the official history of their family.

Von Neumann's experience as a logician enabled him to see at once that computers are logic machines, and to propose the "von Neumann architecture" still incorporated in virtually every modern electronic computer.

A hundred fifty years later, George Boole (almost certainly unaware of Leibniz's efforts along the same lines) produced the first serviceable symbolic logic. More general than Aristotle's logic—which dealt exclusively with syllogisms—Boole's was not yet as versatile as the informal logic mathematicians habitually employ. It remained for his successors to produce a version of symbolic logic that, even in principle, included the entire range of inferential tools employed by mathematicians since the time of Pythagoras and Euclid. Yet Boole's logic was sufficiently powerful to reduce a variety of complex arguments to simple sets of equations, a capability he demonstrated by applying his method to a then notoriously confusing proof—proposed during Leibniz's lifetime by a prominent English philosopher and theologian named Samuel Clarke—of the existence of God. The method is also sufficiently powerful, as Claude Shannon demonstrated in his MIT master's thesis (1938), to simplify the design and analysis of complex switching circuits.

Boole was born in 1815; in 1831, in part because of the failure of his father's business, he became a schoolmaster. During his twenty-year career in primary education, he somehow found time to publish a dozen papers in the *Cambridge Mathematical Journal*, as well as a lengthy paper in the *Philosophical Transactions of the Royal Society*. One of the former, which appeared in two parts in 1842, can be read as founding the study of invariants. Yet Boole published nothing more on the subject, even after his appointment as professor at the newly founded Queen's University in Cork, Ireland. That post came his way because of the professional relationships Boole was able to establish with leading mathematicians of the day. Among them was Augustus De Morgan, who eventually turned Boole's attention away from the algebra of difference and differential operators of his early publications, back toward the study of logic he had begun as a teenager.

In early Victorian England, it was only beginning to be understood that the power of algebra derives from the fact that the symbols representing familiar mathematical quantities and operations combine according to a small number of laws, such as the associative, commutative, and distributive laws. It was only natural to wonder how many other objects and operations could be shown to obey some or all of these same laws, and how much of the newly recognized power was transferable to other objects and operations.

Boole developed an algebra in which variables like x and y could assume only two values: 0 (false) and 1 (true). If x is true only for the elements of a class X, and y is true only for elements of Y, then xy is true (= 1) only for elements of $X \cap Y$. In addition, $1 - x$ is true only for the complementary class $\neg X$, so that $x + (1 - x) = 1$. Everything belongs to either X or $\neg X$. Furthermore, xx is true if and only if x is true, so that $xx = x$, a fact of which Leibniz had also been aware. Manipulating the latter equation in the usual manner yields $x - xx = x(1 - x) = 0$, a triviality in which Boole was delighted to recognize nothing less than Aristotle's famed principle of contradiction: *No object can both belong and not belong to the class of objects possessing a given quality.* His equation proved nothing new, but it gave him confidence that he was on the right track. Boole found logic in more or less the condition in which Aristotle had left it. Since then, the field has evolved almost continuously.

In 1879, Gottlob Frege published a booklet of fewer than 100 pages titled *Begriffschrift*, which Davis describes as a hard-to-translate word constructed from *Begriff* ("concept") and *Schrift* (roughly "script" or "mode of writing"). In it Frege specified a symbolic logic rich enough to include, at least in principle, all the modes of inference ordinarily employed by mathematicians. In doing so, he was not just applying logic to the study of mathematics, but creating a new language. Since he intended algebra, analysis, and geometry to emerge as superstructures resting on the foundation he had undertaken to construct, his new (written) language employed notations unlikely to be confused with those of older and better established disciplines. Among his "sub-mathematical" symbols were the now-familiar $\forall$, $\exists$, $\neg$, $\wedge$, $\vee$, and $\supset$, meaning "for all," "there exists," "not," "and," "or," and "implies."

Davis notes in passing that he was invited to address a scientific meeting held in 1979 to commemorate the 100th anniversary of Frege's *Begriffschrift*, tracing as best he could its consequences for computer science. Davis's preparation for the event was his introduction to the roots of computer science in symbolic logic, and marked the beginning of the research that led to the book under review.

Frege's spirits, like Cantor's, were at times darkened by a lack of contemporary recognition. Neither was ever called to teach in a leading university, and Frege was never even promoted to a full professorship at the University of Jena, where he held forth until his retirement, in 1918. Subsequently, he embraced (and unfortunately recorded in a diary he kept during the last year of his life) a number of the antisemitic and proto-fascist opinions then circulating in Germany. He died in 1925, embittered as much by Germany's defeat during World War I as by his own career disappointments.

The book continues in similar fashion to describe the lives and contributions to symbolic logic of the central figures in that discipline. Leibniz, Boole, Frege, Cantor, Hilbert, Gödel, and Turing rate chapters of their own, while Dedekind, Kronecker, Peano, Russell, Weyl, Brouwer, Church, Cohen, von Neumann, et al. are woven into the text as needed. It is a scholarly work, replete with illuminating facts and appropriate citations. It is also exceedingly well written, and full of informative digressions on the mathematical backdrop against which the critical events took place.

Cantor plays a critical role in the development of logic because of the difficulties encountered by those who tried to incorporate the totality of his transfinite cardinal and/or ordinal numbers into a single set. If there were a set of all cardinal numbers, the cardinality of that set would have to exceed any cardinal number. And so on. The various paradoxes that arose from such attempts led no less a figure than Poincaré to predict that "the Cantorians," should they persist in their attempts to incorporate actual infinities into the body of mathematical knowledge, would be un-able to avoid the sin of self-contradiction.

Davis is particularly interesting on the subject of von Neumann, whose experience as a logician enabled him—according to Davis—to see at once that computers are logic machines, and to propose the "von Neumann architecture" still incorporated in virtually every modern electronic computer. That architecture was suggested by Alan Turing's pre-war vision of a universal computing machine, and Davis is at pains to determine what von Neumann knew of Turing's work, and when he knew it. Davis describes the ENIAC, which was almost complete when von Neumann joined the project in 1945, as an electronic marvel but a logical mess. Its successor, the EDVAC, was a far more successful machine, due to its reliance on the von Neumann architecture.

Both Davis and Chaitin remark, in discussing Gödel's proof, that a large part of the effort went into the construction of the Gödel numbering system for formulas, which closely resembles a computer language like Fortran, the very name of which arose as a contraction of FORmula TRANSlator. It seems unlikely that the similarity would have been apparent to anyone not deeply immersed in both logic and the machine-level programming of computers. However different the two books (and authors) may be, each in its (or his) own way is both thought-provoking and informative.

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