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Preconditioning in Eigenvalue Computations

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How can we catch a lion in a mousetrap? First, we use preconditioning to reduce the size of the lion to the size of a mouse, without changing the size of the trap. Second, we iterate until the lion gets into the mousetrap within the required tolerance.

Preconditioning of a mathematical problem can be informally defined as a transformation of the problem into an equivalent problem that is more suitable for numerical solution. For the iterative solution of a linear system $Ax = f$, textbook preconditioning transforms the system to $BAx = Bf$, where B is a linear operator, called the preconditioner. Here the goal of the preconditioning is to change the system matrix, in particular its spectrum, in order to accelerate the convergence of iterative solvers. For eigenvalue problems $Ax = \lambda x$, however, we need to achieve the acceleration of convergence while limiting changes in the spectrum to those that will allow us to recover, at least approximately, the original eigenvalues of interest.

Mini-workshop participants. Courtesy of Mathematisches Forschungsinstitut Oberwolfach.

The standard technique for accelerating the convergence of classical Krylov subspace eigensolvers is based on spectral transformations, such as shift-and-invert, which emphasize the eigenvalues of interest by moving them to the exterior of the spectrum. New advances in this area include structure-preservation methods and methods for treating nonlinear eigenvalue problems.

Recently, increasing attention has been directed to a more general preconditioning concept, where the classical Krylov subspaces are replaced by subspaces that are not necessarily based on functions of the matrix A. Such preconditioning makes it possible to solve eigenvalue problems with preconditioners developed for linear systems, e.g., multigrid-based techniques; a result is algorithms of linear complexity, where the computational costs grow linearly in the problem size.

The latest accomplishments in this area were presented at a mini-workshop, Preconditioning in Eigenvalue Computations, which took place March 4–8, 2002, in Oberwolfach, in the middle of the Black Forest in southern Germany. The concept of mini-workshops (with fewer than 20 participants), introduced recently at the Mathematical Research Institute in Oberwolfach, was ideally suited for the discussion of bleeding-edge techniques. We found the workshop extremely stimulating and intensive. It has led to a better understanding of what preconditioning means for eigenvalue problems and has generated several new collaborations.

A complete mini-workshop report, containing abstracts of the talks, is available at http://www.mfo.de/Meetings/Documents/2002/10b/Report12_2002.ps.

To reflect the new developments, a special issue of the journal *Linear Algebra and its Applications* will be devoted to the topic of the workshop; see http://www.netlib.org/na-digest-html/02/v02n24.html\#3.

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