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The Accuracy Issue and Numerical Eigenproblems

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The third International Workshop on Accurate Solution of Eigenvalue Problems (IWASEP 3 for short) convened at the Arcadeon Conference Centre in Hagen, Germany, July 3–6, 2000. The first formal gathering of this group occurred as recently as July 1996.

Reinforcing the idea that posters have some clear advantages over talks, the IWASEP organizers used posters for their own presentations and awarded prizes for best poster; shown here accepting first prize is Rui Ralha.

Our group's activities have caught the attention of scholars in Russia, and we were delighted that a significant subset of them joined us in Hagen for the first, but we hope not the last, time: Academician S. Godunov, together with I. Ibraghimov, H.Ikramov, A. Knyazev, E. Biberdorf, and A. Malyshev. With a total of 54 people attending the workshop, some of us feared that the cozy atmosphere of the first two meetings would be lost. On the other hand, the expansion can be seen as an inevitable aspect of growing up. The attractive Arcadeon Centre, where we lived for four days, is situated in a residential district of Hagen, at the east end of the Ruhr valley. There were no urban distractions, but the food was just too good for the modest will power of the participants with whom I spoke. The evening walk was a necessity.

Krešimir Veselić, the managing organizer, received much assistance from the organizer of the first IWASEP, Ivan Slapničar. Their efforts were rewarded by the buzz of excitement and the contented air of most of the participants. As mentioned in reports of the earlier meetings, our criterion for a workshop is that questions be asked on the spot and *not* saved until the end of a talk. As in previous years, there were lively exchanges in most of the sessions.

The increase in numbers obliged us to make use of posters in lieu of some talks. The danger, of course, was that posters tend to be regarded as second-class contributions. To deflate this common perception, the members of the organizing committee all used posters for their presentations, and a prize was given for the best poster. The winners were R. Ralha (1), T. Koprucki (2), and Veselić (3).

All 15 posters were well displayed at the back of our meeting room throughout the four-day meeting, and more than 90 minutes each day were reserved exclusively for viewing and discussing posters. It became apparent that posters have some advantages over talks; in particular, the reader can be sure of understanding the notation. In a meeting as narrowly focused as this one, it might not seem that such a difficulty would arise; with diverse applications on the agenda, however, it certainly does. Other groups might well take note of our innovation: Posters presented by several senior researchers go a long way toward removing the "second-class" stigma. How long will it be before the keynote presentation at some small meeting is a poster?

These workshops, with nearly 100% attendance at every talk, are intense. Several of us were exhausted after four days, despite a relaxed excursion on Day 3 to a "village" of old-fashioned crafts and industry. Now let me turn from format to content.

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What is coming into focus, slowly, is that backward stability—the 40-year-old paradigm that many of us have taught—is not enough. It is a little self-serving to say that with uncertain data we cannot ask for more than the correct answer to a nearby problem. The trouble is that there may be several near-by problems with very different outputs. "Too bad," I usually say, "but the fault is with the data, not with the algorithm. We will give you a condition number if you like." This attitude is defensible but not compassionate.

Consider small unsymmetric perturbations of the identity matrix I. The venerated QR algorithm can easily return an eigenvector matrix that is singular to working accuracy. The algorithm has no way of knowing that the given matrix is close to I, and it is just too glib to say that all perturbations of a given norm are equally valid. It is good to know when the generated

Participants in IWASEP 3, a workshop characterized by nearly 100% attendance at every talk, have grown accustomed to lively exchanges during most sessions.

matrix is close to a "nice" one.

The preceding remarks are what is behind the word "accurate" in the title IWASEP. No one has sold the QR algorithm as an inaccurate solution to the eigenvalue problem. For some of us, however, it has taken nearly 40 years to realize that backward stability is not enough.

The first (1996) workshop focused mainly on multiplicative (or relative) perturbation theory, sophisticated varieties of Jacobi and *J*-Jacobi methods, and the outline of a new approach to the symmetric tridiagonal case. The second meeting (organized by J. Barlow; see *SIAM News*, March 1999) expanded this core syllabus with sessions on applications and software.

At the Hagen meeting the previous themes were developed further, but attention was given to quadratic and even higher order eigenproblems, as well as to so-called iterative methods, where the matrix entries are not available and only a few of the smallest eigenvalues are wanted. Speakers on this topic were P. Arbenz, M. Hochstenbach, U. Holz, A. Knyazev, C. Beattie, A. Ruhe, I. Ibraghimov, T. Koprucki, and K. Neymeyr. Non–self-adjoint problems also came under scrutiny, and Godunov discussed criteria for spectral dichotomy: Are eigenvalues inside or outside the complex unit circle? The point is that to answer this physically relevant question, there is no need to plunge into the—possibly unstable—eigenvalue computation. Instead, more stable Ljapunov objects are computed. Vese-lić's contribution had the same tenor: With semigroups, Ljapunov theory is more reliable than the eigenvalues when a matrix is non-normal.

It was gratifying to the old guard that a significant number of presentations were developments of ideas presented in the first two workshops. B. Grosser ingeniously modified the (holy grail) method of Dhillon and Parlett to obtain orthogonal left and right singular vectors for bidiagonals in $O(n^2)$ effort, however close the singular values. O. Marques described the latest implementation of the dqds algorithm for computing those singular values to good relative accuracy. (The marginal cost of getting those tiny singular values to full precision, instead of resting content with accuracy relative to the largest value, is not more than 5% and is sometimes negative. It is rather weird that demanding more accuracy can result in faster execution, but that is an indication of the fascination of eigenvalue calculations.) A. Malyshev combined the Godunov/Fernando double factorization technique with deflation in order to deal with tightly clustered eigenvalues of symmetric tridiagonals. J. Molera described his work with F. Dopico and Moro, an approach that differs from those of Malyshev and Dhillon/Parlett. The Spanish researchers compute the SVD first and then, through some clever manipulations, attach the correct sign to turn the singular value into an eigenvalue. This strategy seems to yield good relative accuracy whenever the matrix is presented in a factored rank-revealing representation.

In the same "core syllabus" vein, Ralha presented a rival to the techniques for bidiagonal reduction in LAPACK and NAG. Local experts Barlow, J. Demmel, and Z. Drmac were sceptical but intrigued. Let us hope that the issue will be resolved by the next workshop (in 2002). Breaking new ground, N. Higham and F. Tisseur extend the notion of pseudo-spectra (the plural being needed because of the parameter) from the linear case to the polynomial eigenvalue problem with structured perturbations. To capture applications in control theory, it is important to constrain perturbations in order. Higham and Tisseur gave illuminating examples to illustrate the concepts. I. Ipsen gave us perturbation theory in a form that permits absolute and relative formulations with equal ease. Various other flavours of perturbation theory were given by Dopico, S. Rump, I. Slapničar/N. Truhar, T. Stykel, Truhar/Li, and J. Xue. We can conclude that this topic is still hot.

Demmel did not talk about singular values this time but opened our eyes to the inherent difficulty, when the number of bits is included, of assessing the complexity of any highly accurate floating-point computation. D. Watkins, who joined us for the first time, discussed the preservation of Hamiltonian structure for large sparse problems arising in elasticity applications.

Not all contributions were devoted strictly to accuracy, but the accuracy issue appears to be a good catalyzing motto in treating the numerical eigensolution.

A special issue of Linear Algebra and its Applications *will be devoted to the themes of this workshop; J. Barlow, B. Parlett, and K. Veselić are the editors. Further details on the Hagen workshop can be found at* http://www.fernuni-hagen.de/MATHPHYS/veselic/abstracts/1report.html. *The fourth workshop in the series will be held in 2002 in Split.*

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