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At the Intersection of Mathematical Physics And Finance

An Introduction to Econophysics: Correlations and Complexity in Finance. *By Rosario N. Mantegna and H. Eugene Stanley, Cambridge University Press, Cambridge, UK, 2000, 148 + x pages, $34.95.*

What little *The Predictors* (see accompanying review) actually reveals about the concepts and techniques brought by statistical physicists Farmer, Packard, and colleagues to the study of financial time series is presented in almost gossip-column format, with terms like "turbulence," "scaling," and "tick-by-tick analysis" dropped into the narrative like spices into a stew. For those seeking substance, the present volume can serve as a handy technical appendix. Warning: Like most technical appendices, this one is often terse. Readers approaching the contents for the first time should also consult the authors' list of 158 additional references, which range from recent research reports to M.M. Navier's original memoir on fluid motion.

As one of nature's most mysterious macroscopic phenomena, turbulent fluid motion has long been the subject of intense investigation. When pressure gradients are gentle, the atmosphere can flow in a fixed direction past a given point for an extended period of time at an almost uniform rate. But as the pressure gradient grows steeper, the flow undergoes a more or less abrupt transition to turbulence, in which the wind speed is highly variable. Water traveling through a section of pipe exhibits a similarly abrupt transition as the end-to-end pressure difference increases. With modern measurement techniques, detailed records of wind speeds and/or flow rates through pipes can be accumulated and analyzed. The record $V(t)$ of wind velocity measured at a particular place, for a time period $0 < t < T$ during which the Reynolds number remains high, is typically erratic. The corresponding fluid velocity estimate $U_h(t) = V(t + h) - V(t)$, formed for the small fixed h, is typically more so.

When $U_h(t)$ is sampled frequently, the resulting histogram is slightly skewed, markedly leptokurtic, and decidedly fat-tailed, as are the histograms obtained by high-frequency sampling from most financial time series. It is only natural, therefore, that the possibility of earning large sums of money by adapting the models and methods of statistical physics to the study of finance should be vigorously explored. The authors emphasize that, as even the most cursory examination of the historical record reveals, "geometric Brownian motion" is at best a first approximation to the actual movements of the price of any real stock or collection of stocks. Even their assumption that the governing processes are stochastic—rather than examples of deterministic chaos—may in time be disproved by sufficient-ly sensitive measurement techniques.

The book's 15th and final chapter proposes a "random-volatility" (RV) model that reduces to geometric Brownian motion in simple circumstances, and that seems to explain all or most of the irregularities present in actual financial time series. But the authors' RV model lacks many of the most appealing attributes of existing models. Even the celebrated Black–Scholes formula for the evaluation of a (European) stock option has no analog in markets governed by their RV model, owing to the nonexistence of "riskless portfolios" in such markets.

The intermediate chapters form an introduction to non-Gaussian stochastic processes, summarizing vast quantities of physical and market research, and offering brief introductions to (among other things) Lévy stochastic processes, stable distributions, power-law distributions, scaling and self-similarity, and rate-of-convergence results. The contrast between long- and short-range correlated random processes is explained, as are ARCH and GARCH processes, pure jump processes, and something called "truncated Lévy flight." A single seven-page chapter explains arbitrage, the efficient-markets hypothesis, algorithmic complexity theory, and the amount of information in a financial time series.

Chapters 12 and 13 offer an interesting discussion of the multidimensional processes required to mimic the price behavior of an entire portfolio, as opposed to that of a single stock or stock index. While highly correlated pairs of stocks abound, the "anti-correlated" pairs required for the construction of (what are hoped to be) riskless portfolios are harder to find. Indeed, whereas the Standard & Poor list of 500 significant stocks contains in a typical year numerous pairs (i,j) for which the annual price correlation coefficient ρ_{ij} exceeds 2/3, there are seldom if ever any for which $\rho_{ij} < -1/3$.

The authors also define a distance function δ_{ij} between pairs of stocks that turns all such pairs into a metric space, and furnishes an algorithm for turning that space into a more useful "ultrametric" one. An ultrametric space supports a distance function δ_{ij} that satisfies (i) $\delta_{ij} = 0 \Leftrightarrow i = j$ and (ii) $\delta_{ij} = \delta_{ji}$ but substitutes for the familiar triangle inequality the so-called ultrametric inequality

$$\delta_{ij} \leq \max \{\delta_{ik}, \delta_{kj}\}.$$

"Ultrametric spaces," the authors write, "provide a natural way to describe hierarchically structured complex systems, since the concept of ultra-metricity is directly related to the concept of hierarchy." Rather than elaborate, however, they direct the reader equipped with "a background in physical science" to a 1986 paper by Rammal et al. in the *Review of Modern Physics*.

Although parts of the book seem impenetrably terse, it undeniably constitutes a useful introduction to the burgeoning—and highly technical—intersection of finance with mathematical physics.—*James Case.*