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Mathematics in Manufacturing: New Approach Cuts Milling Costs

By Michael Bieterman

Can mathematical methods used in the design and control of aerospace vehicles also be applied to the manufacture of aerospace parts? At The Boeing Company, as described in this article,* the answer is yes.

Improved engineering and manufacturing processes are two of the objectives of The Boeing Company's Mathematics and Computing Technology (M&CT) organization. Some years ago, M&CT researchers Donald Sandstrom and I began looking at ways to improve the generation of tool paths for use in pocket machining.

In pocket machining, material is removed from stock, layer by layer, until pockets are formed and a manufactured part emerges. The tool path for a layer of a pocket is the centerline path along which a tool—an endmill—is fed as its rotating teeth cut the material. Most machining of aerospace parts can be thought of as pocket machining. This is not as surprising as it might seem at first: An aerospace vehicle needs to be both strong and light. Pocket machining, with the support structure it leaves between pockets, helps achieve these properties.

In the aerospace industry (and elsewhere), tool-path generation is done with NC (numerical control) programming, in which a person provides a part description and feed-rate requirements to a computer-aided manufacturing software package. The CAM package then produces a file with path and feed-rate data that are postprocessed into a form a controller can use to drive a machine tool. Conventionally, the data are specified pointwise. More and more commonly, however, splines are used for the smooth representation and compression of data that vary along a tool path.

Our objective was to improve on what CAM packages currently deliver by developing a fairly general tool-path generation method that would reduce the time required to machine a pocket. The main part of the method, as described below, uses geometry input and a partial differential equation that arises in many engineering design applications to generate a path. The path generation is the focus of this article.

The second (supporting) part of the method is a process for determining a feed rate that varies along the path, taking advantage of both the shape of the path and the capabilities of the cutting machine used. This process consists of solving a trajectory optimization problem of a type that arises in flight control. The algorithms use a smooth parametric representation of the path and various machining constraints.

As verified by computational and aluminum-cutting experiments, our new method for generating tool paths reduces machining time by up to 30% as compared with conventional methods. It also greatly extends tool life when hard metals are being cut, as we saw in titanium-cutting experiments. For the future, we expect considerable reductions in machine-tool wear and tear over time as paths generated by the new method are used. There are potential economic benefits in such results, and patent protection has been sought.

Defining the Problem

Our investigations were performed in the context of a high-speed machining project. Such a setting involves high spindle speeds for rotating tools and high axis drive velocity and acceleration/deceleration capabilities for feeding tools. How high is high-speed? A relevant answer for machining is a lot like that for high-speed computing: high enough to change underlying assumptions and paradigms.

One of the pocket-machining paradigms changed by high-speed machining is that machine dynamics can be ignored when a tool path is generated. Traditionally, the best path was the shortest, and it reflected the part shape (pocket boundary shape) in its entirety.

In a high-speed setting, by contrast, the best path must account for machine dynamics. It is a longer path, and away from the pocket boundary it sometimes resembles the part shape only vaguely. Such a path turns out to be very advantageous in many conventional machining situations as well. The advantages increase with the ratio of the distance required to stop a tool to the pocket length.

Figure 1 shows a conventional parallel-offset tool path used to machine a rounded-corner pentagonal pocket. In the construction of such paths, the pocket boundary (dashed line in the figure) is offset inward a fixed distance, and the process is then repeated. The pocket is typically machined outward. Starting at the pocket center, the tool moves sequentially along each path orbit about the center and then goes on to the next orbit.

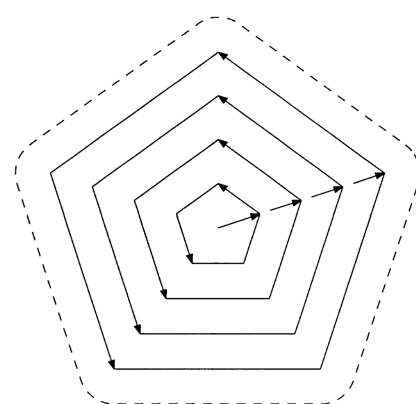

Figure 1. *Conventional tool path for pocket machining.*

*The article is a modified version of a portion of the author's talk at the SIAM Northwest Regional Mathematics in Industry Workshop, held at the University of Washington, Seattle, in October 2000; see sidebar for more about the regional workshop series.

The main problem with this and related conventional tool paths is that they inherit any corners or regions of high local curvature in the part. The result is a limit on how fast a tool can be fed. Coming into a corner, the tool has to decelerate to rest, or near rest, only to accelerate as it comes out of the corner. Why, we wondered, limit the feed rate of a high-speed machine in this manner? Analogously, what would be the point of buying a fast sports car for stop-and-go and turn driving through city traffic lights?

The path corners can be rounded slightly, which helps a bit. However, all of a tool's acceleration capability would still be concentrated in tight-radius corners, rather than being distributed along the path where it could decrease total machining time.

Cornering also contributes greatly to tool wear when hard metals are cut. It is not known exactly why this is the case. Some mechanical wear occurs when a tool hits a corner. Perhaps more importantly, however, the increase in temperature that occurs in corners as the tool's engagement with the stock suddenly increases could contribute significantly to tool wear.

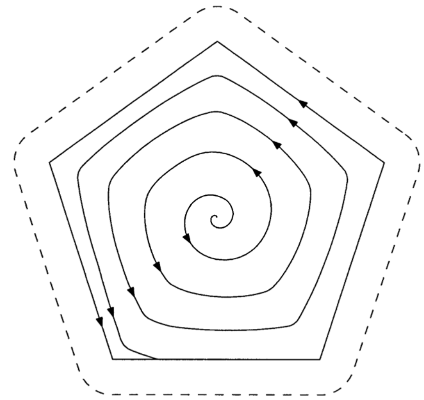

Figure 2. *Curvilinear tool path for pocket machining.*

Practicing machinists and NC programmers knew *what* to do to solve the problem: Introduce sweeping curves over much of the path. What evidently was not known before in the NC programming and CAM communities was exactly *how* to do this automatically for fairly general pockets in a way that would require much less machining time.

Solution: Slow Morphing Into the Part Shape

One approach would be to pose and attempt a formal solution to an optimization problem. This sounds good, but the tool-path machining time minimization problem with path, feed-rate, acceleration, and other types of constraints is much harder even than the traveling salesman problem. The optimization problem is extremely difficult even for pockets that are convex (or not *too* nonconvex), that are free of islands or pad-up regions, and that are machined by removal of constant-depth layers. The vast majority of pockets of interest at Boeing have these properties.

Our new approach for generating curvilinear tool paths for such pockets centers on the approximate solution of an elliptic PDE boundary value problem that appears in many engineering design analyses. The problem is defined on a two-dimensional pocket region whose boundary is offset inward, by one tool radius, from the part.

We solve the eigenvalue problem approximately for the Laplacian

$$-\nabla^2 u = \lambda u,$$

subject to Dirichlet boundary conditions $u = 0$, and the normalization

$$\max_{(x,y)} u(x, y) = 1$$

for the principal eigenfunction u (corresponding to the smallest of the positive eigenvalues $\{\lambda_j\}$).

Elliptic PDE operators have nice smoothing and positivity properties, which are just what we needed in our tool-path generation problem. The constant-value contours—or level sets—of the principal eigenfunction typically have progressively less local maximum curvature as the constant varies from zero to one (i.e., as the distance inward from the boundary increases). The positivity, or maximum principal, for the PDE makes it possible to generate a tool path as we do—that is, by beginning at the location where the solution u takes its maximum value and then continuously spiraling outward.

The spiraling takes place between appropriately spaced contours of u until all material has been removed. The spacing is determined by a user-specified maximum stepover, or width of cut. Figure 2 shows a curvilinear tool path generated with this method for the pentagonal pocket shown in Figure 1. Unlike the conventional path pictured in Figure 1, the spiral curvilinear tool path is nearly circular at the pocket center and slowly morphs into the part shape as it gets closer to the part.

Figure 3 is a recent photograph of a test part with four pockets (and with coolant and chips strewn about) being machined by the curvilinear tool-path method. To some extent, in the slight impressions left on the pocket floors, the figure shows the similar morphing of the spiral tool paths. This is because endmills always leave signatures of their paths on the floors of the metal pockets they cut.

Our initial choice of solving an eigenvalue problem was not critical. Other elliptic PDEs with appropriate positivity would work too (e.g., replacing λu by a constant in the PDE). We chose the eigenvalue problem partly because of wave-guide analogies that came to mind, and also because of the possibility that our results could be extended to much more complex parts. For such parts, it would probably be preferable to decompose the pockets into smaller pieces rather than to use a single spiral path that repeatedly traversed some parts of a pocket. One way to decompose a pocket automatically would be to use nodal lines ($u = 0$ contours) of one of the higher-order Laplacian eigenfunctions.

For readers interested in learning more about mathematical aspects of tool-path problems for pocket machining, [3] is a good starting point. Details of the present curvilinear tool-path method and constrained feed-rate optimization procedure are documented in two Boeing-proprietary technical reports [1,2]. We are preparing manuscripts containing most of this information for publication in the open literature.

Meanwhile, a few more details on the present methods, with some additional illustrations, were included in a recent presentation I made at the Institute for Mathematics and Its Applications at the University of Minnesota; see [4].

My colleagues and I are currently working to extend the curvilinear tool-path method so as to alleviate some of the restrictions mentioned in this article. We are also involved in the process of transitioning this technology for production use. But these are stories for another article.

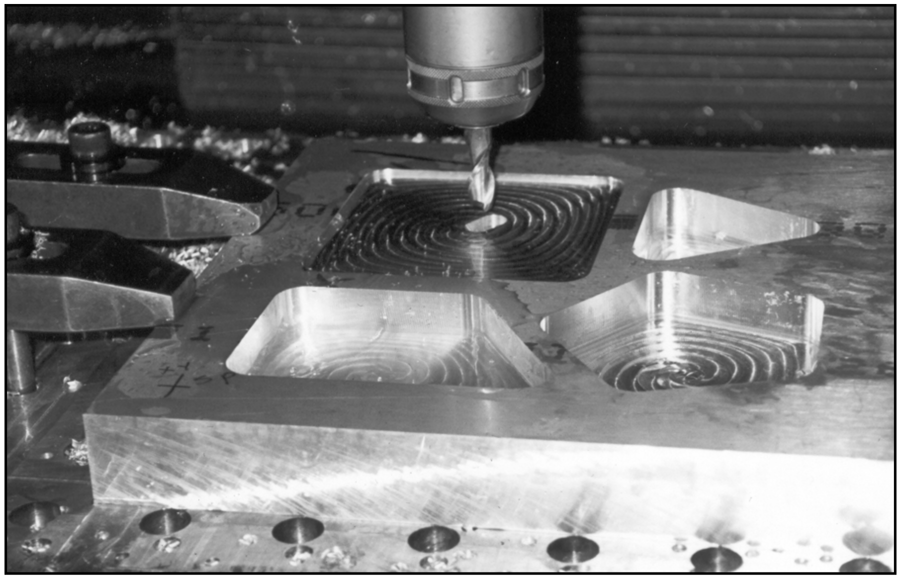

Figure 3. *Recent test of the curvilinear tool-path method.*

Acknowledgments

Among the many people who contributed to the work described here are Don Sandstrom, who co-invented the curvilinear tool-path method and recently retired from Boeing; Tom Grandine and Jan Vandenbrande of Boeing M&CT; and Paul Wright and his group in the Mechanical Engineering Department at the University of California, Berkeley, where metal-cutting demonstrations of our tool-path method were first performed.

References

[1] M. Bieterman and D. Sandstrom, *Curvilinear spiral tool path trajectories for high-speed pocket machining*, Tech. Rep. SSGTECH–98–031, Boeing Proprietary, December 1998.

[2] M. Bieterman and D. Sandstrom, *A strategy for prescribing high-speed pocket machining tool paths*, Tech. Rep. ISSTECH–96–020, Boeing Proprietary, September 1996.

[3] M. Held, *On the Computational Geometry of Pocket Machining,* in *Lecture Notes in Computer Science*, 500, Springer-Verlag, New York, June 1991.

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Second Phase of SIAM's MII Project: Six Regional Workshops

Industrial and other nonacademic employers, SIAM pointed out in its 1995 *Report on Mathematics in Industry* (www.siam.org/mii), are not interested in hiring a mathematics graduate whose credentials are limited to expertise in a single area of mathematics. What these employers are looking for, rather, is flexible scientists with broad, interdisciplinary backgrounds, computing skills, the ability to communicate their work to a wide variety of audiences, and a genuine interest in their organizations' missions.

Having released the report, which includes recommendations for mathematics departments seeking to improve the preparation of students for nonacademic careers, SIAM moved on to the next phase of the MII project: a series of regional workshops. The workshops—a total of six, geographically distributed throughout the U.S. and funded by the National Science Foundation—brought together faculty, especially those working to develop new curricula, people in industry who hire mathematics graduates, and students. Each workshop had a unique charac-ter, defined mainly by the nature of the industry in the region; with presentations of industrial programs in Mexico and Canada by participants from those countries, the workshops became a North American series.

The series began in the Northeast with a May 1998 workshop at Worcester Polytechnic Institute (see Paul Davis's report in *SIAM News*, September 1998; www.siam.org/siamnews/09-98/wpi.htm). Speakers at the WPI workshop provided suggestions for university departments interested in initiating interactions with industry, along with a look at the types of problems considered in such interactions.

From WPI, the series moved to the Midwest, with a workshop held at the University of Illinois, Chicago. Drawing on local industry, that workshop was able to offer perspectives from a wide range of industrial environments, with representatives from financial, communications, pharmaceutical, and manufacturing concerns. (Paul Brown's informative tale of his search for nonacademic employment, which began as a banquet speech at the workshop, appeared in modified form in *SIAM News* in June 1999; www.siam.org/siamnews/06-99/track.htm.)

Subsequent MII workshops were held in the West (Claremont, June 1999; see report in *SIAM News* in September 1999; www.siam.org/siamnews/09-99/claremont.htm), the Southeast (North Carolina State University, October 1999), the Northwest (University of Washington, Seattle, October 2000), and the Southwest (Houston, April 2001). The Claremont meeting, again reflecting local resources, featured mathematics as applied in Hollywood, satellites, and bioengineering. At NCSU, the emphasis was on curriculum development; energy and the environment constituted a major theme at the Houston workshop. The accompanying article by Michael Bieterman of The Boeing Company was drawn from his presentation at the Seattle workshop.

"By the time of the later workshops," says SIAM technical director William Kolata, "it became clear that new programs were emerging at many universities to serve graduate students with an interest in nonacademic employment."

In the course of these workshops, Kolata continues, faculty from about 120 universities, large and small, were able to exchange information among themselves and with their industrial counterparts in more than 50 companies; participating faculty obtained ideas for new curricula, along with suggestions for establishing contacts with industry as a source of employment opportunities, research collaborations, and new problems.