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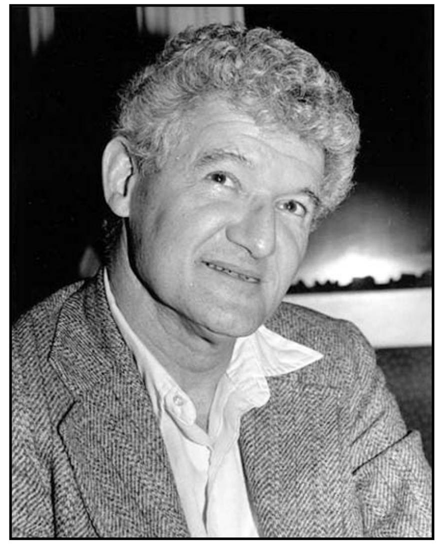
Peter Lax to Receive 2005 Abel Prize

By Gilbert Strang



Citing "groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions," the Norwegian Academy of Science and Letters announced that Peter Lax will receive the 2005 Abel Prize in Mathematics. King Harald of Norway will present the prize to Lax in Oslo on May 24. A professor emeritus at the Courant Institute of Mathematical Sciences, New York University, Lax is the third recipient of the annual prize, which this year carries a cash award of approximately $980,000.

King Harald of Norway will present the 2005 Abel Prize in Mathematics to Peter Lax on May 24. The annual prize, awarded this year for the third time, was created by the Norwegian Parliament to honor Niels Henrik Abel. The Web site www.abelprize.no tells the story of the prize and its recipients and Abel himself, but these paragraphs in *SIAM News* are different. They are more personal and mathematical. For many of us, the work of Peter Lax and his encouragement have transformed our subject and our lives.

My own experience began in Peter Henrici's small seminar at UCLA. Each student was asked to report on a recent paper. By chance, I was assigned the 1956 Lax–Richtmyer paper on the stability of linear finite difference equations. That paper proved a theorem, and set a research direction, that became the foundation for this part of scientific computing:

- The Lax Equivalence Theorem established that stability is necessary and sufficient for the convergence of a consistent finite difference method.
- The von Neumann test for stability (on each e^{ikx}) requires boundedness of a family A^n of powers of matrices.

The equivalence of stability and convergence is the Fundamental Theorem of Numerical Analysis. Its proof involves a neat application of functional analysis, to show that stability is necessary. Unstable difference methods have solutions $A^n e^{ikx}$ with $|A(k)| > 1$, but this pure exponential can still converge for each k (because A approaches 1 as the mesh size goes to 0). What we need is a single initial function, combining different k's, for which the discrete solution blows up. Constructing that $u(x,0)$ would not always be fun, but we don't need to do it. The Uniform Boundedness Theorem says it exists, unless there is a fixed bound on all the A^n that appear.

For systems of linear equations, in which A is a matrix, the stability test opened a beautiful problem in matrix analysis. Heinz-Otto Kreiss solved that problem in 1959. He found that a bound C on the norm of $(|z| - 1)(zI - A)^{-1}$ for $|z| > 1$ implies that A^n is bounded by KC. The proof is now polished to the point that we know K exactly: e times the matrix size n.

Beyond this theory comes the inventive part of our subject, to construct algorithms that are stable and accurate and fast. That is fun! Peter Lax did his full share, beginning with the Lax–Friedrichs and Lax–Wendroff schemes. LF uses a positive combination of u_{i-1} and u_{i+1} at step n, to construct u_i at step $n + 1$. Stability is safe, but the accuracy is low. By including also u_i at step n, LW takes the crucial step to second-order accuracy. Positivity is lost. But stability still holds for mesh ratios that satisfy the von Neumann test.

The real (ongoing) challenge is solving nonlinear hyperbolic systems with shocks and "fans." Here, Peter Lax's great contribution was to identify, among many possible solutions, the right one. From gas dynamics he extracted an "entropy condition" that applies much more generally: Information travels along characteristic lines *into* shocks, never out. The shock travels at a speed that is between the linearized wave speeds on its two sides.

Core Courant faculty and close friends Peter Lax and Louis Nirenberg. Each, on receiving a PhD at NYU in 1949, stayed on at the then newly named Courant Institute of Mathematical Sciences, where decades later each would serve as director, Nirenberg (1970–72) succeeded by Lax (1972–80).

Capturing that shock with finite differences has been an adventure. The names of Glimm, Godunov, Harten, Osher, and Roe appear (with Lax and many others!) in books like Randy LeVeque's. Finite volume methods conserve the physics as they discretize the problem. Discontinuous finite elements have also made headway, using Runge–Kutta in time. This competition of ideas is the reality of scientific computing, and it is terrific.

Among other nonlinear partial differential equations, a few magical ones are "completely integrable." The best known is $u_t + uu_x = u_{xxx}$, named after Korteweg and deVries (KdV). It has traveling wave solutions $u(x - ct)$ called "solitons" that pass each other and unexpectedly keep their shape (as Kruskal and Zabusky saw on the screen). That is possible because the integrals of u and u^2 and infinitely many less obvious quantities are constant in time. The puzzle was to understand those conserved quantities in other completely integrable equations too.

Lax's solution is truly beautiful. He rewrote the equations as $L' = BL - LB$. For matrices L and B, the solution $L(t) = \exp(Bt)L(0)\exp(-Bt)$ always conserves the eigenvalues of $L(0)$. For nonlinear operators, a Lax pair L,B is again the key. (The Schrödinger op-erator $Lv = -v'' + uv$ produces KdV with the right B, which is just amazing.) This is the gentle touch of mathematics, illuminating the whole problem, and we admire it when we see it.

I cannot end without remembering tennis in Central Park with Peter and his wonderful son Johnny. Occasionally there was an invitation from Peter and Anneli to stay overnight. You may know that Peter never forgets the experience of working with von Neumann. In our turn, many of us remember so well how our lives were changed by this great mathematician who has won the Abel Prize.

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