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What's So Special About Zero?

Zero: The Biography of a Dangerous Idea. *By Charles Seife, Viking Penguin, New York, 2000, 248 pages, $24.95.*

As an analyst trained at Princeton University, I was weaned on Fefferman's Principle: Every constant is equal to one. The intention is that if you are proving an estimate—in harmonic analysis, let us say—the value of the constant is irrelevant to the validity or utility of that estimate. So you may as well declare all constants to be equal to one. The single constant that must be excepted from Fefferman's Principle is the number zero. There is no way to normalize zero and turn it into one.

BOOK REVIEW

By Steven G. Krantz

If I were the author of the book under review, I would consider the last sentence of the preceding paragraph to be a revelation. Because what I am really saying is that one divided by zero equals infinity and, in Charles Seife's own words,

> Zero is powerful because it is infinity's twin. They are equal and opposite, yin and yang. They are equally paradoxical and troubling. The biggest questions in science and religion are about nothingness and eternity, the void and the infinite, zero and infinity. The clashes over zero were the battles that shook the foundations of philosophy, of science, of mathematics, and of religion. Underneath every revolution lay a zero—and an infinity.

The trouble with this book is that it is difficult to tell what it is *about.* Absent the title, I could page through the text and conclude that I was reading a book about the history of counting (Chapter 1), or complex analysis (Chapter 6), or the discovery of calculus (Chapter 5), or general relativity and quantum mechanics (Chapter 8), or the limit concept (Chapter 2).

What is so special about zero? I could easily argue that 84 is a very important number, because it comes up in Hurwitz's celebrated formula about the number of automorphisms of a compact Riemann surface of genus g. Or that 2π is very important, because it comes up in virtually every formula in Fourier analysis and complex analysis. Or that 4 is important, because it is the least dimension for which we know the Poincaré conjecture to be true.

There are precisely two important properties of zero:

- It is the additive identity in the integers.
- Its presence in various number systems makes those systems closed under the prevailing operations.

Lurking behind every example in Seife's book is one of these two properties. The truth of this assertion will not always be apparent to the reader, just because Seife has a difficult time sticking to the point. Chapter 1 professes to be a discussion of the ancient history of zero but wanders off into discourses on nuclear submarines, ancient methods of counting, Babylonian and Mayan number systems, chaos, and modern logic. Chapter 8 goes on at length about general relativity and quantum mechanics and claims that "zero lives where the two theories meet" (page 192). What does this mean? I was not even aware that the two theories had been properly introduced.

Chapter 8 gives a rather slipshod description of what string theory is about. Seife claims that the theory is probably useless (and must be classified as philosophy rather than physics) because

- There will not in the foreseeable future be any experiment that can confirm or deny the existence of strings.
- Strings live in ten-dimensional space.
- Zero is lurking around in there somewhere (though he is hard put to say just where).

In the second point, Seife is paying intellectual tribute to John Horgan, the author of *The End of Science* [1]. It is Horgan, a writer of impeccable scientific credentials (i.e., a journalist and erstwhile English major from Yale), who taught us that the entire scientific enterprise has come to a grinding halt and that, as evidence of his bold assertion, string theory must be nonsense because it is modeled on ten-dimensional space. (I am reminded here of Marilyn vos Savant's "proof" [4] that Andrew Wiles's solution of Fermat's last theorem must be incorrect—Wiles's proof uses hyperbolic geometry, and the circle can be squared in hyperbolic geometry, which is well known to be impossible.)

The history of zero really is quite interesting, and what is fascinating about it to a serious scholar is the epistemological aspect of the subject. Before the invention of modern number systems, people had a difficult time conceiving of the need for a number denoting zero. If we accept that a "number" is an equivalence class of sets (i.e., all the sets with that number of elements), then it is something of a leap to form the analogous equivalence class for sets with no elements. And even if one has found a way to conceptualize zero, then one must find some way to write it down, and to treat it like any other number. Although Seife hints at these issues in Chapter 1, he never really comes right out and discusses them. He is too busy claiming that every scientific phenomenon or puzzle—from mechanics to relativity to string theory—is explainable in terms of the zero concept. I may as well make my fortune by writing a book about one. After all, take anything in the universe and multiply it by one and what do you get?—Well, Katie bar the door, you get that object right back again!

Most of this book is based on the following logical canard: The assertion $2 + 2 = 5$ is, after some elementary manipulation, equivalent to division by zero. The assertion $\int x^2\,dx = x^4$ is, after a bit more effort, equivalent to division by zero. In fact, any false statement is, on a formal level, equivalent to division by zero. When Charles Seife tells us that a nuclear submarine failed because the software was attempting to divide by zero, or that the lack of a unified field theory can be blamed on the fact that quantum mechanics and general relativity confront each other over zero, or that string theory is flawed because of the way it addresses zero, what, then, is he really telling us? Not much, I am afraid.

By his own admission, John Horgan bases his theories about the demise of modern science on an idea of the literary critic Harold Bloom of Yale University. To oversimplify, Bloom thinks that the works of modern writers are all derivative of the works of the old masters (Milton, Chaucer, etc.). In like manner, Horgan thinks that Newton and Kepler had all the great ideas; scientists today are just mopping up. Of course science is not literature, and it is not clear that a paradigm that may apply to the latter (though I would challenge Bloom to prove that Allen Ginsburg's *Howl* or James Joyce's *Finnegans Wake* is derivative) will also apply to the former. There will never be another scientist like Newton because, for all practical purposes, Newton was the first great theoret-ical scientist. A similar statement could be made about Kepler vis à vis astron-omy. Scientists today are unlike Newton and Kepler because science is operating on a different plateau, building on the ideas of the great historical masters.

I do not have much use for Horgan and his theories, but I will say this: Horgan enun-ciates a thesis, argues a point, and draws a conclusion. He knows what he is trying to say and he says it. I cannot give the same credit to Charles Seife. Seife's text is anecdotal; he seems more drawn to the desire to amuse than to inform. He wants to tell a story. I am not sure that the story-telling mode suits mathematics well.

We mathematicians are quite accustomed to the notion that modern ideas build on old ones, and that the old ones (once proved valid) never become invalid. As a result, we tend to expect the books we read both to acknowledge and to de-velop that *Weltanschauung*. The book under review, unfortunately, does not do so. If I were to pick a single word to describe Seife's book, it would be "phenomenological." The paradigm *modus ponendo ponens* plays no role in his book. It is all "point and see." As a professional mathematician, I find a book written in this manner difficult to understand.

Seife's enthusiasm sometimes gets the better of him. On page 115 he presents Isaac Newton's original calculation of the derivative of $y = x^2 + x + 1$, gleefully declaring that "Newton's method of differentiation doesn't look very much like the one we use today." In point of fact, Newton's methodology is *precisely* the one we use today—we simply use a different notation, and we allow the limit concept to play a more prominent role. I am afraid that some of the other renditions of scientific "fact" presented in this book are equally careless.

I have gone to some lengths to criticize Seife's book, so let me now praise it a bit. It is quite difficult to explain mathematics to the layperson. If you think you are going to tell the butcher what the Seiberg–Witten equations or the Bieberbach Conjecture is about, then woe is you. Authors wishing to attempt this daunting task have two choices: They can speak in generalities, or they can choose simple topics. In the book under review, Seife does both. My view is that you can tell the truth about zero and still make your writing accessible to the layperson. Seife's choice is different: He wants to wax poetic and wander all over the modern Gestalt, in an effort to give the reader a feeling for his subject. This is not the way I would do it, but it is a worthy effort.

If people read Seife's book and come away with a positive feeling about mathematics, then so much the better. If such readers then decide to read another book about mathematics (perhaps something more substantial, like Körner's *Joy of Counting* [3]), I will be ecstatic. I am troubled that the reader of Seife's book will get a cockeyed impression of what the mathematical enterprise is all about. As mathematicians, we know that mathematics is not sociology, or phenomenology, or even epistemology. It is a rigorous form of discourse for seeking certain types of truths. The nonmathematical reader of Seife's book simply will not learn this basic fact from these pages.

Even so, Seife's effort has merit. He writes well, and his prose is fun to read. The book is a careful piece of work—it is nicely documented and well illustrated (unlike, for example, the other popular book about zero [2] that is sweeping the market these days). I would have liked to see more scholarly—and fewer popular—references in the bibliography, but that is just me—I am a pedant. I would have liked Seife to be more scrupulous about sticking to his subject matter, and not to fall so easily into the episodic mode. I would have liked to see the book organized around ideas, rather than around the quips that this author uses as chapter titles. I would have liked to see more depth and less fluff.

But Seife is not writing for me, and he is probably not writing for you. He is doing a job that most of us do not know how to do, which is to communicate *something*—*anything*—about mathematics to the public. I can only hope that the reading of this book will give people a positive impression of the fascination of our subject.

References

[1] J. Horgan, *The End of Science*, Broadway Books, New York, 1997.

[2] R. Kaplan, *The Nothing That Is: A Natural History of Zero*, Oxford University Press, Oxford, 1999.

[3] T.W. Körner, *The Pleasures of Counting*, Cambridge University Press, Cambridge, 1997.

[4] M. vos Savant, *The World's Most Famous Math Problem*, St. Martin's Press, New York, 1993.

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