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Using Wavelets for a 3-D Geometric Squeeze

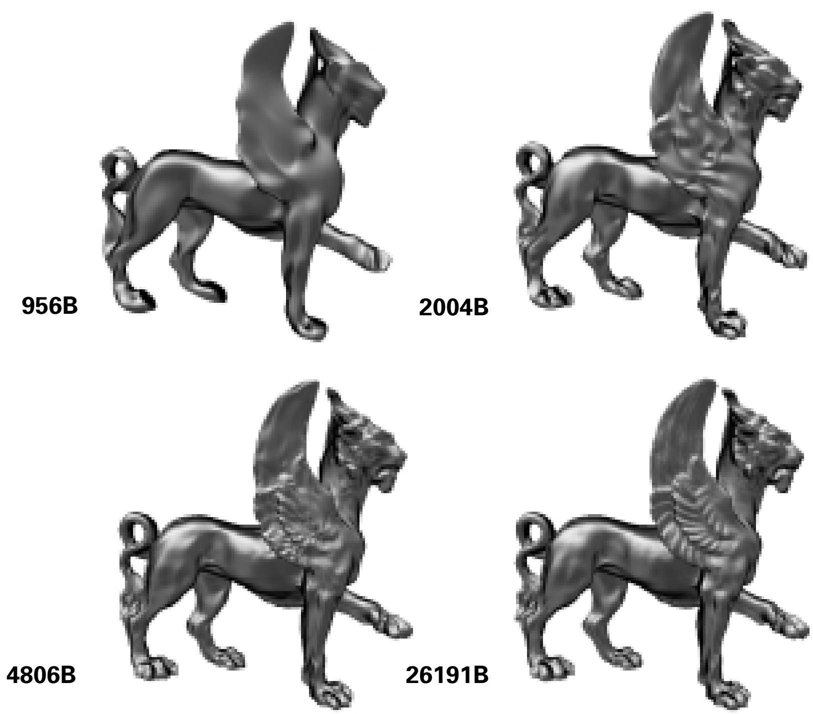
By Barry A. Cipra

Contrary to the impression you might get from surfing the Internet, the real world is three-dimensional. To date, Web designers have been frustrated in their efforts to make realistic, easily down-loadable 3-D graphics. But a new wavelet-based algorithm may change all that. A team of computer scientists, led by Wim Sweldens of Lucent Technologies' Bell Labs and Peter Schröder of Caltech, together with postdocs Igor Guskov and Andrei Khodakovsky, has developed a practical way to deal with data describing arbitrarily complicated geometric surfaces. They presented their method, called progressive geometry compression, at the SIGGRAPH 2000 conference, held July 23–28 in New Orleans.

Fast, high-quality 3-D graphics have obvious applications, ranging from medical imaging to computer games. The new technologies could make the traditional police mugshot a relic of the last century. Other promising uses include "virtual parts" catalogs of manufactured items and on-line custom tailoring. E-tailers will undoubtedly find myriad ways to exploit the third dimension.

"Think of real estate," Sweldens says. "Today someone selling a house puts pictures of all the rooms on the Web. Soon the seller may be putting a video walkthrough of the house on the Web. When geometry processing reaches the desktop—in software like today's digital photo and video editors—you'll not only be able to see any view of any room in the house, but you'll also be able to see how it will look after you knock out a wall, repaint the rooms, and drop in new furniture from a 3-D catalogue."

Three-dimensional graphics itself is hardly new. In principle, all you need is a sufficiently refined mesh—a triangular mesh, say, with three coordinates per vertex and a bookkeeping system to keep track of faces and edges. Subdivision can take care of any smoothing that may be required—if, for example, you want to see how light reflects off a surface. Producing such graphics is also well within current technology; laser scanners are already capable of taking detailed 3-D pictures and digitizing the results.



Sequence of subdivision renderings of a winged cat (file sizes in bytes). Progressive geometry compression can handle surfaces with arbitrary topology, such as that produced by the creature's tail.

The problem is, you get a lot of damn digits: Modern meshes can have millions, or even billions, of vertices. "It's just a vast amount of data," says SIAM president Gilbert Strang. The sheer quantity "is a very difficult barrier to surmount."

The challenge is to store and transmit these enormous data sets using as few bits as possible. The same problem, of course, arises in the 1-, 2-, and (2+1)-dimensional applications of audio, images, and video. But the straight line and flat plane (video being a succession of planar images) lend themselves to old-fashioned Fourier analysis. Curved and crinkled surfaces, on the other hand, do not. It's not that Fourier coefficients can't describe such complicated objects; it's just that way too many are needed to do a decent job.

Enter progressive geometry compression.

The key idea is that the locations of almost all the vertices of a dense 3-D mesh can be described by a single real number per vertex instead of the obvious three. The trick is to tailor the mesh with this one-for-all/all-for-one efficiency in mind.

The basic idea can be explained one dimension down, with the description of a curve in the plane (see Figure 1). For a coarse polygonal approximation, with each vertex lying on the curve, each segment is simply divided in half, with the midpoint moved back to the curve along a line perpendicular to the segment. Because the original location of the midpoint can be computed from the coordinates of the segment's endpoints, the new location can be described by the distance moved (with a positive or negative sign used to indicate the direction of the move).

In three dimensions, triangles play the role of line segments (see Figure 2). The subdivision cuts each triangle into four by connecting the midpoints of the three sides. Again, each midpoint is relocated to the surface, with a single real number describing the distance moved. The only new wrinkle is to decide what direction should play the role of "perpendicular," but there's a natural candidate: Each midpoint belongs to two triangles, each with a normal vector (though Möbius strips and Klein bottles cause extra aggravation), so it makes sense to use their sum.

In practice it may be desirable (or necessary) to move the midpoints somewhat sideways as well. Sweldens and his colleagues refer to information of this type as "parametric" data, as distinct from the "geometric" data of the normal component, which, they observe, contains the bulk of the 3-D information. The final data type is the "connectivity" of the mesh, which is largely built into the subdivision process. In particular, most of the vertices are connected to six other vertices.

With this approach, coarse-mesh data can be used to produce a quick sketch of a complicated surface, which can then be refined and re-refined, much as a fuzzy picture is brought into focus.* This is the progressive part of the technique.

The geometric information itself can be compressed, with wavelet techniques. Sweldens has developed a new method, called "lifting," that is geared for computing wavelet transforms of geometric data. A key idea is that the wavelet coefficients are best associated not with vertices of the finer mesh, but with edges of the coarser mesh. The reason, Sweldens explains, is that when a pair of adjacent triangles in the coarser mesh are subdivided, four of the new edges stay roughly parallel (see Figure 3). The researchers report that their technique for geometry compression is 6 to 12 times more efficient than previous methods.

Much work remains to be done, they point out, including extensions of the method to higher-dimensional data sets. But for now at least, according to Sweldens, "geometry is poised to become the fourth wave of digital multimedia communication." Gentlemen, start your search engines.

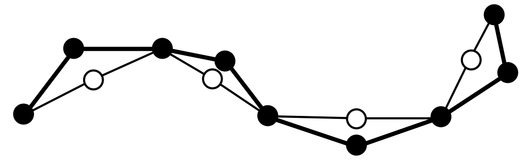


Figure 1. *A coarse polygon (light lines) is refined by subdividing at the midpoints.*

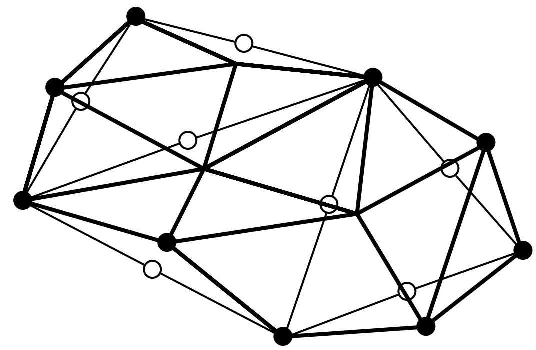


Figure 2. *A coarse mesh (light lines) is refined by subdividing at the midpoints.*

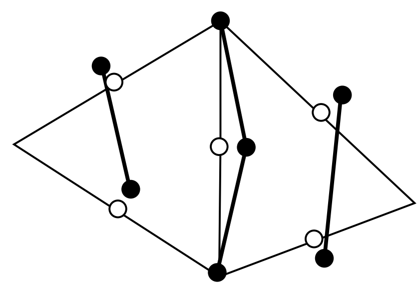


Figure 3. *If the midpoints didn't move, the dark lines would be precisely parallel.*

*See the winged cat sequence on page 1, which is from the paper "Progressive Geometry Compression," by Sweldens and colleagues. The paper, which can be accessed at http://cm.bell-labs.com/who/wim/papers/compression, is a good resource for readers who would like to learn more; another is the paper "Normal Meshes," at http://cm.bell-labs.com/who/wim/papers/normalmesh.

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