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Yet Another Meeting on Matrix Computations?

By Beresford Parlett

A group of about 30 specialists congregated in the conference center of Pennsylvania State University from July 20 to 23, 1998, for the Second International Workshop on Accurate Solution of Eigenvalue Problems. (For this group, it should be pointed out from the start, the term "eigenvalue problems" includes the singular value decomposition.) Half of the attendees had spent an intense four days together two years earlier in the captivating city of Split, Croatia. Whence the need for more meetings on matrix computations? Are there not enough already?

The title of the workshop might seem to be "loaded." Does the group wish to distinguish itself from those lowly individuals who compute inaccurate solutions? The answer points in the opposite direction. A shared goal among numerical analysts is to compute approximations "as accurate as the data warrant." In most situations the data do not define their output (eigenvalues and eigenvectors) very well, and it is pointless to strive for lots of correct digits. We should be grateful for small normwise backward errors. On the other hand, there are situations in which highly relatively accurate eigenvalues allow faster computation of the eigenvectors.

In the last ten years there has been limited but growing interest in special situations in which some or all of the eigenpairs are defined by their data to relatively high levels of accuracy. It has thus seemed reasonable to ask whether there are methods that will compute such eigenpairs to full accuracy. It was natural for those conducting research along these lines to seek kindred souls who could then work together to clarify the nature of the new enterprise.

The group's first workshop focused on "relative" perturbation theory, Jacobi methods, and the dqds algorithm, with just a glance at indefinite problems. These themes were continued in July 1998, but the scope of the meeting had become broader. Both workshops were characterized by frequent questions during presentations and none of the feeling (as in bigger meetings) that interruptions must be curtailed in order to preserve the time table. That is the great reward of workshops: Experts can pursue topics in depth when they arise.

The hard-core participants, in attendance at both meetings, were Jesse Barlow, James Demmel, Zlatko Drmac, Ilse Ipsen, Saša and Sanja Singer, Ivan Slapnicar, Françoise Tisseur, Kresimir Veselic, and the author. The few who dropped out after the first were outnumbered by the new faces, some of whom are mentioned in the following account of the second workshop. Jesse Barlow, who took responsibility for organizing the second workshop, deserves most of the credit for its success. In particular he brought in new people who served to link the group's interests to other work, both theoretical and practical.

In a superb tutorial on how crystals lead to operators with gaps in their essential spectra, Christopher Beattie raised the issue of determining when matrix eigenvalues are associated with continuous spectra and when they are approximating eigenvalues of the operator. He went on to give a new perspective on Temple–Lehmann bounds, which are intimately related to harmonic Ritz values. Beattie's approach reveals that these approximations have natural duals that have hitherto been ignored, perhaps because they are not so easy to compute.

On the practical side, we were lucky to have the active participation of Louis Komzsik, who has been in charge of the NASTRAN library's numerical software for 20 years. He led a discussion of software and then, in his formal presentation, talked about the quadratic eigenproblem and its difficulties. In particular, he reported that a relatively high degree of accuracy (about one third of the precision length) is required in his problems. Tisseur addressed backward error analysis for the quadratic problem in her presentation.

David Stewart described another problem that was new to most of the group: How do you compute the singular values of a long product of small matrices? The difficulty is that different Lyapunov exponents lead to singular values whose ratios decline exponentially.

In another excellent tutorial, Hongyuan Zha discussed some aspects of Latent Semantic Indexing (LSI) computations.

Przemyslaw Kosowski, who had just finished his doctoral dissertation at the University of Warsaw, pointed out that standard Sturm–Liouville theory somewhat contradicts our experience with symmetric tridiagonal matrices: Clusters of close eigenvalues do not occur "naturally"; the higher eigenvalues, moreover, are not well determined by the standard coefficient functions. There is room for more work on this anomaly.

Space restrictions permit only brief highlights from the talks of the hard-core members:

- Barlow described his high-accuracy reduction to bidiagonal form.
- Demmel described work he has done with several collaborators (some of whom were at the meeting) on obtaining high-accuracy SVD via decompositions of the form XDY^T, where X and Y are simply well conditioned and D is a diagonal scaling matrix.
- Drmac showed some clever ways of making Jacobi SVD competitive in arithmetic effort with rival techniques. The benefits from the point of view of accuracy were discussed at the first workshop.
- Ipsen gave a clear account of when relative perturbations can be related to conventional (additive) perturbations.
- Both Slapnicar and Veselic gave new results on indefinite eigenproblems of the form

$$G^TG - \lambda J,$$

where $J = \mathrm{diag}(\pm 1)$ and the original matrix $H = GJG^T$. The so-called spectral absolute value of H, which can be expressed as $G\,G^T$, plays a fundamental role. Juan Molera, one of the new participants, gave an excellent talk on this same topic.

■ The author gave lots of pictures of his and Inderjit Dhillon's new algorithm for finding orthogonal eigenvectors (of symmetric tridiagonals) without using Gram–Schmidt. A high degree of relative accuracy is essential here. Included in the presentation were some theoretical results for highly relatively accurate small eigenvalues of LDL^T determined by the entries of L and D.

The hard-working attendees took a break one afternoon for a picnic near Penn's Cave, followed by a boat ride through the cave.

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