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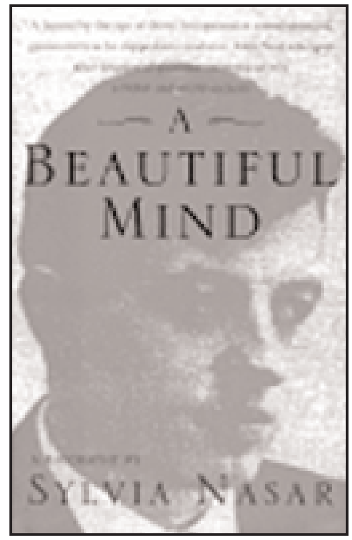
The Life and Times of One of the Century's Most Gifted Individuals

A Beautiful Mind. *By Sylvia Nasar, Simon & Schuster, New York, 1998, 459 pages. $25.00.*

BOOK REVIEW

By James Case



The author, best known as an economics correspondent for *The New York Times*, describes her biography of John Nash as a play in three acts: genius, madness, and reawakening. It is hard to say exactly when, during the 1950s, it became clear that Nash was a rare genius. He began the decade as a promising graduate student at Princeton and ended it in madness. In the interim, he did the work he will always be remembered for, in game theory, geometry, and analysis. His reawakening from madness is no easier to date than his descent into it but seems to have been well under way by 1990, according to those who saw him on a more or less daily basis in and around Princeton. He spent the intervening decades in varying degrees of mental illness, the depths of which Nasar strives mightily to plumb.

The book devotes only a chapter to Nash's boyhood in Bluefield, West Virginia, and another to his undergraduate years at the Carnegie Institute of Technology—now Carnegie Mellon University—which he attended on a Westinghouse scholarship between June 1945 and June 1948.* Nash arrived well prepared, having completed numerous courses at Bluefield College while still in high school. His friends from those years remember him as a "brain," destined to become a "scientist," presumably of the white-coated Hollywood variety then making relentlessly publicized contributions to the war effort. He excelled in school but displayed no particular affinity for any one subject. Socially, he is remembered as awkward and immature.

At Carnegie, after enrolling as an engineering student, he switched almost immediately to chemistry and somewhat later to math. The regimentation of mechanical drawing class was not to his taste, and he was chastised for breaking chemistry lab equipment. His second year found him studying tensor analysis and relativity with mathematics department chair J.M. Synge, ranking among the top ten (though not the top five) in the Putnam competition, and being urged by his instructors to consider a career in mathematics. By his third (and final) year at Carnegie, he was the star performer in a class in quantum mechanics taught by R.J. Duffin from the original (German-language) version of von Neumann's book; his classmates included Hans Weinberger, Raoul Bott, and one or two others. Harvard, Princeton, Chicago, and Michigan—then (Nasar reports) the nation's top four graduate programs in mathematics—offered scholarships.

In college, even more than in high school, Nash's peers found him weird, immature, and socially inept. The scholarship students at Carnegie were housed together in a single dormitory and took most of their classes together as well-small ones taught by carefully selected instructors. For the first time ever, the members of this hand-picked elite found themselves among kindred spirits. "We were all nerds back in our high schools," recalls Weinberger, who roomed with Nash for one (and only one) term, "and here we were able to talk to one another." Yet Nash remained a misfit. As classmate Paul Zweifel recalls, "We sensed that he had a mental problem. . . . We tormented poor John." Nasar points out that Nash had a lot going for him in those days—size, good looks, physical strength, and unmistakable talent—and suspects that envy was an incitement to the hazing he endured.

The Non-cooperative Theory of Games

During his final semester at Carnegie, needing to fulfill a distribution requirement, Nash enrolled in a course in international trade taught by an expert on cartels and trade agreements. It was the only formal training in economics he was ever to receive. The ink was barely dry on the Bretton Woods agreement of 1944, designed to prevent a repetition of the debacle of the 1930s, during which international commodity markets had been hamstrung by cartels, monopolies, and---above all---a tangled web of bilateral trade agreements. While learning about such things, Nash began to formulate what he now calls "the general bargaining problem." With such problems already in mind, he was naturally drawn---on arrival in Princeton---to A.W. Tucker's seminar on game theory, where he could try out his ideas on the participants. Oskar Morgenstern found the ideas meritorious and urged publication.

Eager to make his mark at Princeton, Nash was anything but reticent. His contemporaries remember him with more clarity than fondness. Martin Shubik, a fellow resident of the graduate college, reports that "Nash was spiteful, a child with a social IQ of 12." Yet talented. Nasar could find no one who recalled ever seeing him with a book. He appeared to spend his time walking the cor-

*U.S. colleges and universities operated on an accelerated year-round schedule during World War II, designed to produce graduates after only three years in residence.

ridors of Fine Hall or lying, lost in thought, on tables in the library and commons room. Eugenio Calabi suspects that Nash, like himself, was marginally dyslexic. But while Calabi regarded his own aversion to reading—especially dense, challenging material—as a weakness, Nash would defend not reading on the ground that mathematical activity would trump mathematical passivity in the long run.

"It was," recalls John Milnor, "as if he wanted to rediscover, for himself, three hundred years of mathematics." "More than any other student I have known," Norman Steenrod was soon to write, "Nash believes in learning a subject by doing research in it."

Nasar reports Tucker's surprise when, during the summer of 1949, Nash asked him to direct his thesis. Initial expectations were that Nash would work with Steenrod in algebraic geometry. About to depart for a sabbatical year at Stanford, and thinking that he would be back long before there was any real directing to do, Tucker quickly agreed. Little did he know that Nash had already confided his ideas about "equilibrium points" to his classmate David Gale, who was sure they contained a thesis. Nor was Tucker aware that Nash had already run his idea past von Neumann, only to be told "That's trivial, you know. That's just a fixed-point theorem."

Indeed, Nash's proof that every finite multiplayer game has at least one equilibrium point represents a direct generalization of von Neumann's original (fixed-point) proof of the min-max theorem for matrix games. Both theorems have since been proved by more direct methods, usually involving convexity. Nasar omits the story, frequently told by Morgenstern in later years, that von Neumann had based his own initial attempt to construct a theory of many-player games on just such a theorem. But Morgenstern, being familiar with the equilibrium-point concept from the 19th-century writings of A.A. Cournot, had been ready with an objection. At length, and with some difficulty, he managed to persuade von Neumann that the Cournot–Nash approach was too superficial to resolve the enduring economic issues raised by the Great Depression—issues involving coalitions, cartels, cooperation, and (above all) trade unions.

Once persuaded, von Neumann apparently stayed persuaded, and doubted the worth of Nash's rival "non-cooperative" theory of games. On the other hand, Nasar adds, the preface to the final (1953) edition of von Neumann and Morgenstern's instant classic *The Theory of Games and Economic Behavior* makes a point of directing the reader's attention to Nash's work.

Three Landmark Results

After receiving his degree in June 1950, at the age of 21, Nash returned to the problems in the geometry of manifolds on which he had begun work with Steenrod. Except for the work of a few summers at the RAND Corporation, his contributions to game theory were complete.

Manifolds were a hot topic during the 1950s, due mainly to their connection with relativity. Nash conjectured, and was eventually able to prove, that any smooth, compact n-dimensional manifold M is diffeomorphic to some connected component V_0 of a real algebraic variety $V \subset \mathbb{R}^{2n+1}$. To most, the result seemed too strong: Whereas smooth manifolds were deemed plentiful, real algebraic varieties (being the solution sets of systems of polynomial equations) appeared scarce. It came as a real surprise to the mathematics community that each of the former was actually one of the latter, albeit in disguise.

A short time later, while working as an instructor at MIT, Nash was challenged by a colleague to decide whether it is possible to embed an arbitrary Riemannian manifold in Euclidean space. The question, first posed explicitly by Ludwig Schläfli in the 1870s, was subsequently mentioned in passing by Hilbert, Cartan, Weyl, and others. But, as Nash observed in his 1995 Nobel autobiography, "This problem, although classical, was not much talked about as an outstanding problem. It was not like, for example, the four-color conjecture." He solved the problem in two stages, proving first that one could "crumple" an arbitrary Riemannian manifold into three-dimensional Euclidean space, if one were willing to destroy its smoothness. Later, he showed that any such manifold could be embedded—smoothly and without self-intersections-in a higher—dimensional Euclidean space.

Armed with a Sloan fellowship, and a sabbatical from MIT, Nash elected to spend the 1956–57 academic year at the Institute for Advanced Study. But with Einstein dead, von Neumann dying, Gödel no longer active, and Oppenheimer humbled by the McCarthy inquisitions, he found the Institute less exciting than expected. He took to spending a good deal of his time at the newly created Courant Institute, the members of which seemed genuinely pleased to have him around. Tilla Weinstein, a mathematician at Rutgers, told Nasar that "He was just a delight. There was a wit and humor about him that was thoroughly un-standard. There was a wonderful playful quality, a lightness." Cathleen Morawetz, whose father (J.M. Synge) had helped to spark Nash's interest in math at Carnegie, found him "very charming," "an attractive fellow," "a lively conversationalist."

At Courant, Louis Nirenberg suggested that Nash try to extend certain estimates—obtained in two dimensions during the 1930s—to higher-dimensional systems. His eventual success received more publicity than his embedding results and convinced researchers in yet another field that Nash was indeed a genius. The year ended in disappointment, however, when it was learned that Ennio De Giorgi, of Pisa, had published equivalent results, in a particularly obscure European journal. Nasar describes the modus operandi by which Nash obtained all three of his landmark results. In each case, he used an eminent sounding board: D.C. Spencer at Princeton, Norman Levinson at MIT, and Lars Hormander at Courant. And in each case, he began by proposing an entire sequence of obviously deficient proofs. Yet each failed attempt was quickly followed by another more persuasive effort, until a valid demonstration emerged. His mentors were invariably impressed by his originality, persistence, and geometrical intuition.

A Missed Fields Medal

Nasar considers it a very real possibility that, had his champions on the selection committee been only a little more persistent, Nash might have won a Fields medal in 1958, along with topologist René Thom and number theorist Klaus Roth. Indeed, she quotes Peter Lax, a former student of committee member K.O. Friedrichs, who was privy to his mentor's feelings on the matter. According to Lax, "He [Friedrichs] was upset. As I look back, he should have insisted that a third prize be given." But Nash was not yet 30, and there was no apparent urgency to recognize his growing list of achievements.

As fate would have it, 1958 was to be Nash's last chance to win. The Fields medal is awarded only once every four years, always to persons under the age of 40; the prize charter stipulates that the honor should "encourage young mathematicians" and "future work." By 1962, Nash's descent into madness was well known and the prospect of future work virtually nil.

According to Nasar, his deteriorating mental condition became unmistakable during a series of seminars he gave in early 1959, concerning the Riemann hypothesis. A standing-room-only audience at Columbia, and a smaller one at Yale, found the lectures to be embarrassing, incoherent, almost stream-of-consciousness monologues containing no new insights or concrete proposals. Soon afterward, he sent a letter declining a chair at the University of Chicago, on the ground that he was about to be named Emperor of Antarctica. Another letter, addressed to Claude Berge in Paris and written in four colors of ink, complained that his career was being ruined by aliens from outer space. By May of 1959, his wife, Alicia—assisted by various MIT officials—had him temporarily committed.

John Nash in December 1994, lecturing at the University of Uppsala a few days after receiving the Nobel Prize in economics. It was only after considerable infighting, as described in the book under review, that the Nobel committee elected to award the 1994 prize to Nash's "non-cooperative" theory of games alone, and not to include work on von Neumann and Oskar Morgenstern's original "cooperative" theory. Photograph from A Beautiful Mind.

Nasar chronicles the pressures and disappointments that may have triggered Nash's illness, and the decades he spent in or between institutions for the insane. Interviews with his younger sister, Martha, the few contemporaries with whom he continued to communicate, and various caregivers supply the bulk of the information from which Nasar extracts her remarkably complete account of Nash's "lost years." Two of Nash's own observations seem particularly revealing: He found his delusions hard to deal with because, he once said, they came to him in the same way that his mathematical ideas did. And, in time, he learned to deal with those delusions by placing himself on a sort of permanent diet—one that directs him to resist the temptation to think about world government and other (to him) appealing subjects.

The Nobel Prize in Economics

The last of the five sections in Nasar's book deals with Nash's Nobel prize (1994), and the infighting that led to it. She has managed, through interviews with several participants, to piece together a plausible account of the process. There were, according to her informants, six names on the short list from which Nash, Reinhard Selten, and John Harsanyi were eventually chosen. The others were Lloyd Shapley and Robert Aumann, both mathematicians by training, and Thomas Schelling, a social scientist. The committee eventually decided that the award should be given for contributions to Nash's "non-cooperative" theory of games alone, rather than shared with developers of von Neumann and Morgenstern's original "cooperative" theory, since it was the former that had "proven fruitful for economics." The prize they were awarding, after all, was "The Central Bank of Sweden Prize *in Economic Science* in Memory of Alfred Nobel" [italics added].

Nasar also describes the controversy surrounding the economics prize itself. It was created in 1968, over heated and formidable opposition within the Royal Swedish Academy of Sciences. Many members still believe that the creation of a sixth Nobel prize was a mistake. They find it disturbing that economic ideas dart in and out of fashion for roughly the same reasons that women's hemlines rise and fall, with no clear evidence of any growing amalgam of theory and fact with which a reasonable person might find it difficult to disagree. Nasar quotes an economist within the academy to the effect that:

> "This is not really a Nobel Prize. It should never be spoken of together with the other prizes. The academy should never have accepted the prize in economics. I have been against the prize since I became a member of the academy."

Were it not widely shared, such an opinion would not be worth repeating. Nasar maintains that it is widely shared, and that calls for abolition surface periodically. What Nasar does not explain is why Nash's "upstart" non-cooperative theory has "proven fruitful for economics," while von Neumann and Morgenstern's more abstract "cooperative" theory has not. The alleged fruitfulness of Nash's theory has very little to do with any progress on the economic questions raised by the Great Depression and less with any perceptible increase in the accuracy of "economic impact statements." It has everything to do with generalizations of a result originally obtained by Cournot in 1838, according to which non-cooperative competition "converges"—as the number of competitors becomes large—to something like the "perfect competition" postulated by Adam Smith. The Cournot–Nash approach to competitive issues is simply less revolutionary, when viewed from the standpoint of traditional economic theory, than the alternative proposed by von Neumann and Morgenstern.

All in all, Sylvia Nasar has written a remarkable—and aptly named book. It does a commendable job of chronicling the personal and professional life and times of one of the century's most gifted individuals.

James Case is an independent consultant who lives in Baltimore, Maryland.