

Omega: Chaitin's Demon

Meta Math!: The Quest for Omega. By Gregory Chaitin, Pantheon Books, New York, 2005, 240 pages, \$26.00.

Problem: Name a book that combines mathematical history, philosophy, more than a whiff of theology, personal palaver, and brilliant insights, along with evidence of a Borges-like imagination, eyebrow-raising mathematical constructions, breathtaking excitement, grandiose ruminations, and some bosh.

Solution: The book under review. This solution may not be unique, but with the addition of a side condition—that the book be a guide for readers seeking to learn where the oracular mathematical demon Omega resides—we certainly have uniqueness.

We live in an age when some of our most brilliant scientists and mathematicians want to go beyond the confines of their disciplines as currently pursued and create something revolutionary: one super-transcendental step for mankind, something meta, and, with an eye on sales, something sensational. Some years ago Gregory Chaitin, one of the stars at the IBM T.J. Watson Research Center, defined a real number (or, more precisely, a construction, because a specific number emerges only after a Turing machine and an encoding have been arbitrarily selected) that he dubbed Ω . Definition: Ω is the probability that a random program on a universal Turing machine will halt under the probability distribution that a program of length k has probability 2^{-k} . In its ambiguity, Ω is something like the god Jupiter—now a swan, now a bull, depending on the opportunities available.

BOOK REVIEW

By Philip J. Davis

Chaitin and his followers claimed remarkable oracular things about Ω . The selection of the name Ω was thought out carefully. The word “omega” puts me—and I suppose many readers—in mind of the biblical verse “I am the alpha and the omega, says the Lord, who is, who was, and who is to come” (Revelation, 1:8). Martin Gardner, enthusiastic expositor of popular mathematics, thus wrote in a 1979 article in *Scientific American*, “The random number Ω bids fair to hold the mysteries of the universe.”

How's that for a puff? The Final Key to the Whole Cosmic Contraption? If you can believe that, you can believe anything, as the Duke of Wellington once remarked. The number Ω is now known somewhat less theologically and apocalyptically as “Chaitin's Constant,” and a value of it can be found in the *CRC Concise Encyclopedia of Mathematics*, as well as in *The Online Encyclopedia of Integer Sequences*. There you will find a decimal version: 0.078749969978123844 Given the ambiguities involved, this value strikes me as pointless, unless some digital kabbalists want to have a go at the segment.

But as I said, Ω is not really a number; it's a definition or a process or a demon. Yes, I would locate Ω among the great demons of physics and mathematics: Laplace's Demon can predict the future location of every particle in the universe using Newtonian mechanics; Maxwell's Thermodynamic Demon can separate out the high-velocity molecules in a container of gas. For all I know, there might be a Quantum Demon who, knowing things forbidden by the Uncertainty Principle, could produce a few small surprises in the laboratory. I always thought that a Diagonalizing Demon (now resident in every text on set theory) helped Georg Cantor write down every existing number on a list and still find another number not on the list. As Sol Feferman pointed out, all non-computability results are ultimately based on the method of diagonalization applied to idealized computers. Now in Ω we have something even more amazing: Ω is Chaitin's Demon, and it can reveal the answers to all mathematical questions, past, present, and future.

Assuming that we've pinned Ω down to any of its manifestations, can we say something about it that is more along traditional and less along halting-theoretic lines? Well, like π , Ω is a transcendental number, i.e., it's not the root of an algebraic equation with integer coefficients. It is a b -normal number, for all bases b ; if, for example, $b = 10$, all n -tuples of successive decimal digits of Ω occur with the appropriate frequency 10^{-n} . Unlike π , whose digits can be generated by a very short program and is thus not Chaitin-random, the digits of Ω are random (whatever random means). A direct consequence of the insolvability of the halting problem is that more than a finite number of bits of Ω cannot be calculated. Consequently, Ω is non-computable. Moreover, it is incompressible, i.e., its sequence of digits cannot be expressed more compactly than the number itself. Thus, Ω “can be known of, but not known.” As with Macbeth's dagger:

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Come, let me clutch thee
I have thee not, and yet I see thee still.

I should therefore describe Ω as ineffable—an adjective that is most often applied to the name of God but that is also popular among the theorists of “large cardinal numbers.”

The concept of Ω resides in the axiomatic theory of computation or in algorithmic information theory, of which Chaitin is a master, and it has a venerable and distinguished genealogy. One can surely cite Zeno, Leibniz, Borel, Gödel, Turing, and Shannon among its forefathers. More recently, it is allied to deep results of Julia Robinson, Jones and Matiyasevic, and others, relating to the number of integer solutions to polynomial equations in many variables.

I first came across “oracular” numbers in the writings of Émile Borel (*Les nombres inaccessibles*, 1952), who pointed out that if the idea of infinite decimals is agreeable to you, you can construct a real number that contains all possible texts. With a bit of indexing you could recover from it, say, Tolstoy’s *War and Peace*, or the 18th Amendment to the U.S. Constitution. Borel considered this number a bit of a joke. I wouldn’t call it a mathematical joke, but I would place it among the mathematical comedies—somewhat akin to the Banach–Tarski Paradox, wherein a pea can be decomposed and reassembled to be the size of the sun. Whatever it is, Ω has spawned a fair number of follow-up papers and so in this sense is hardly a joke.

Although I suspect that there are no theorems in computation theory associated with Ω that cannot be more transparently phrased without it, Ω obviously interests Chaitin greatly. Ω is a sensation, and the desire for sensationalism infects us all. Ω also interests me, and for some of the same reasons. Why? To me, Ω is interesting as an imaginative production that gets me thinking about many points of the philosophy of mathematics. It leads to or connects with questions of ontology, with what a representation is, with when we “know” something, with the conflict between the real and the ideal. It gets me thinking about the tremendously useful mathematical oxymoron known as the definition of randomness. It leads to the old question of a deterministic universe. My own feeling is that the universe—which includes my great satisfaction at having on October 15, 2005, eaten a whole can of King Oscar Brisling Sardines—cannot be described by anything less than the universe itself. Therefore, by a definition of randomness deriving from Martin Löf, Kolmogoroff, et alii, the universe is a random affair. This offends the sensibilities of many people—vanity? spirituality?—including Einstein and those who espouse the notion of Intelligent Design. It leads to a philosophy of limitation for mathematics—and perhaps this is really the point Chaitin wants to hammer home in *The Quest for Omega*.

But at the end of the day, Ω leads me to turn 180 degrees away from these classical philosophical questions—discussed ad nauseam and with diminishing profit—of why mathematics is true, whether its objects and constructs have ontological validity, whether it is the only mode of inference, what its limitations are, whether it is the unique language in which theoretical physics must be formulated. If we could focus instead on mathematical pragmatics—why mathematics throughout the millennia has been useful or deleterious to society—then I believe that we might be able to illuminate an aspect of mathematics that is often ignored: mathematics as a social enterprise, for that is surely what it is.

Meta Math! is full of one liners that are ripe for collecting in a Wit and Wisdom of Gregory book. Here are a few, with my comments attached:

“To understand something is to make it mathematical.” Nonsense. This is the view of a mathematical imperialist.

“The computer is even more revolutionary as an **idea**, than it is as a practical device that alters society.” This is pure platonic idealism, the theory that what we see around us is only a shadow of a higher truth.

“We cannot stick with a single FAS (formal axiomatic system). FAS’s are a failure.” “Theorem proving algorithms do not work.” “Experimentation is the only way to prove that software is correct.” I like these three obiter dicta.

“I don’t think that you can really understand a mathematical result until you find your own proof.” I agree, provided that I can rephrase it as “until you find your own way of looking at the matter and that way agrees with many other people’s way.”

“If something important is true, there are many reasons that it is true.” Right on! That’s the basis of my personal views on what I call mathematical evidence.

“Mathematics needs more axioms.” Hmm. Why? To provide post hoc legitimacy for a certain concatenation of symbols that have been written down? Do we need axioms for a unicorn?

Surf a copy of *Meta Math!* Sit in the lotus position and contemplate Jorge Luis Borges’s Aleph—that place where all other places in the world co-exist—located at the alphabetic antipodes from Omega. And then think about this: Now that we have computer networks that we cannot model à la Turing, a new super Ω may be in the offing that could—wow—out-meta the meta!

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