

Gene Golub SIAM Summer School 2012 –
Simulation and Supercomputing in the Geosciences

SWE – An Education-Oriented Code to Solve the Shallow Water Equations

Michael Bader, Alexander Breuer

Technische Universität München



Teaching Parallel Programming Models ...

Starting Point: Lecture on Parallel Programming

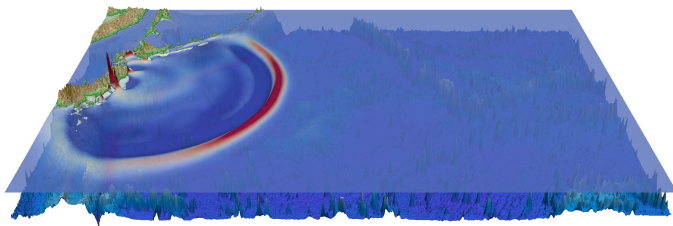
- classical approaches for shared & distributed memory: OpenMP and MPI
- “something more fancy” → GPU computing (CUDA, e.g.)
- motivating example to teach different models and compare their properties

“Motivating Example”:

- not just Jacobi or Gauß-Seidel
- not the heat equation again ...
- inspired by a CFD code: “Nast” by Griebel et al.
- turned out to become shallow water equations
- **and then there was: G2S3!**



Towards Tsunami Simulation with SWE

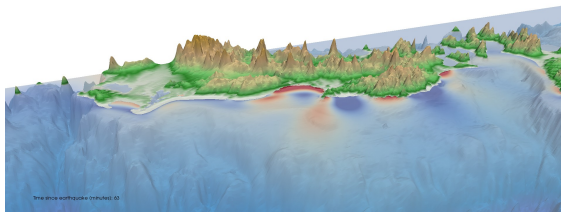


Time since earthquake (minutes): 78

Shallow Water Code – Summary

- Finite Volume discretization on regular Cartesian grids
→ simple numerics (but can be extended to state-of-the-art)
- patch-based approach with ghost cells for communication
→ wide-spread design pattern for parallelization

Towards Tsunami Simulation with SWE (2)

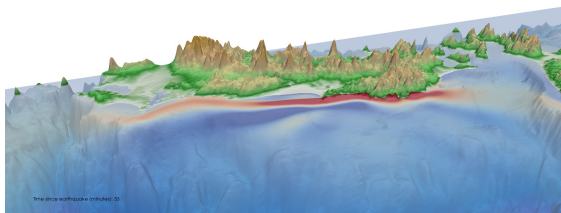


Shallow Water Code – Bells & Whistles

- included augmented Riemann solvers
→ allows to simulate inundation
(George, 2008; Bale, LeVeque, et al., 2002)
- developed towards hybrid parallel architectures
→ now runs on GPU cluster

Part I

Model and Discretization



Model: The Shallow Water Equations

Simplified setting (no friction, no viscosity, no coriolis forces, etc.):

$$\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{bmatrix}_x + \begin{bmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix}_y = 0.$$

Finite Volume Discretization:

- generalized 2D hyperbolic PDE: $q = (h, hu, hv)^T$

$$\frac{\partial}{\partial t} q + \frac{\partial}{\partial x} F(q) + \frac{\partial}{\partial y} G(q) = 0$$

- wave propagation form:

$$\begin{aligned} Q_{i,j}^{n+1} = Q_{i,j}^n & - \frac{\Delta t}{\Delta x} \left(\mathcal{A}^+ \Delta Q_{i-1/2,j} + \mathcal{A}^- \Delta Q_{i+1/2,j} \right) \\ & - \frac{\Delta t}{\Delta y} \left(\mathcal{B}^+ \Delta Q_{i,j-1/2} + \mathcal{B}^- \Delta Q_{i,j+1/2} \right). \end{aligned}$$

Model: The Shallow Water Equations

Simplified setting (no friction, no viscosity, no coriolis forces, etc.):

$$\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{bmatrix}_x + \begin{bmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix}_y = 0.$$

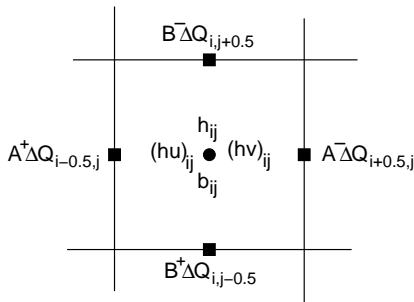
Flux Computation on Edges:

- wave propagation form:

$$Q_{i,j}^{n+1} = Q_{i,j}^n - \frac{\Delta t}{\Delta x} \left(\mathcal{A}^+ \Delta Q_{i-1/2,j} + \mathcal{A}^- \Delta Q_{i+1/2,j} \right) - \frac{\Delta t}{\Delta y} \left(\mathcal{B}^+ \Delta Q_{i,j-1/2} + \mathcal{B}^- \Delta Q_{i,j+1/2} \right).$$

- simple fluxes: Rusanov/(local) Lax-Friedrich
- more advanced: f-Wave or (augmented) Riemann solvers (George, 2008; LeVeque, 2011), no limiters

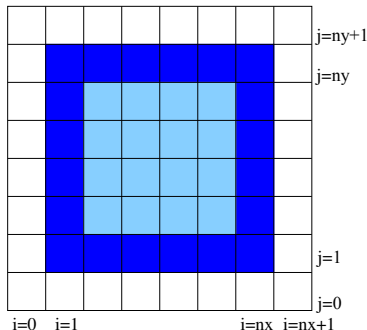
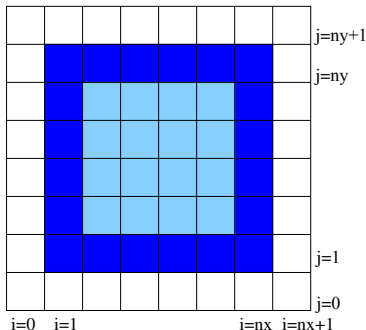
Finite Volume Discretization



Unknowns and Numerical Fluxes:

- unknowns h , hu , hv , and b located in cell centers
- two sets of “net updates”/numerical fluxes per edge:
 $\mathcal{A}^{+}\Delta Q_{i-1/2,j}$, $\mathcal{B}^{-}\Delta Q_{i,j+1/2}$, etc.

Patches of Cartesian Grid Blocks

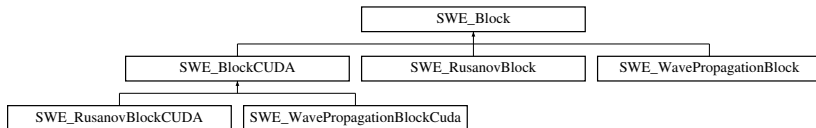


Spatial Discretization:

- regular Cartesian meshes; allow multiple patches
- ghost and copy layers to implement boundary conditions, for more complicated domains, and for parallelization

Part II

Implementation



Main Loop – Euler Time-stepping

```
while( t < ... ) {  
    // set boundary conditions  
    splash.setGhostLayer();  
  
    // compute fluxes on each edge  
    splash.computeNumericalFluxes();  
  
    // set largest allowed time step:  
    dt = splash.getMaxTimestep();  
    t += dt;  
  
    // update unknowns in each cell  
    splash.updateUnknowns(dt);  
};
```

→ defines interface for abstract class **SWE_Block**

Set Ghost Layers – Boundary Conditions

Split into two methods:

- `setGhostLayer()`: interface function in `SWE_Block`, needs to be called by main loop
- `setBoundaryConditions()`: called by `setGhostLayer()`; sets “real” boundary conditions (WALL, OUTFLOW, etc.)

```
switch(boundary[BND_LEFT]) {
```

```
  case WALL:
```

```
  {
```

```
    for(int j=1; j<=ny; j++) {
```

```
      h[0][j] = h[1][j];    b[0][j] = b[1][j];
```

```
      hu[0][j] = -hu[1][j]; hv[0][j] = hv[1][j];
```

```
    };
```

```
    break;
```

```
  }
```

```
  case OUTFLOW:
```

```
  { /* ... */
```

(cmp. file `SWE_Block.cpp`)

Compute Numerical Fluxes

main loop to compute net updates on **left/right edges**:

```
for(int i=1; i < nx+2; i++) {  
    for(int j=1; j < ny+1; j++) {  
        float maxEdgeSpeed;  
        wavePropagationSolver.computeNetUpdates(  
            h[i-1][j], h[i][j],  
            hu[i-1][j], hu[i][j],  
            b[i-1][j], b[i][j],  
            hNetUpdatesLeft[i-1][j-1], hNetUpdatesRight[i-1][j-1],  
            huNetUpdatesLeft[i-1][j-1], huNetUpdatesRight[i-1][j-1],  
            maxEdgeSpeed  
        );  
        maxWaveSpeed = std::max(maxWaveSpeed, maxEdgeSpeed);  
    }  
}
```

(cmp. file SWE_WavePropagationBlock.cpp)

Compute Numerical Fluxes (2)

main loop to compute net updates on **top/bottom edges**:

```
for(int i=1; i < nx+1; i++) {  
    for(int j=1; j < ny+2; j++) {  
        float maxEdgeSpeed;  
        wavePropagationSolver.computeNetUpdates(  
            h[i][j-1], h[i][j],  
            hv[i][j-1], hv[i][j],  
            b[i][j-1], b[i][j],  
            hNetUpdatesBelow[i-1][j-1], hNetUpdatesAbove[i-1][j-1],  
            hvNetUpdatesBelow[i-1][j-1], hvNetUpdatesAbove[i-1][j-1],  
            maxEdgeSpeed  
        );  
        maxWaveSpeed = std::max(maxWaveSpeed, maxEdgeSpeed);  
    }  
}
```

(cmp. file SWE_WavePropagationBlock.cpp)

Determine Maximum Time Step

- variable `maxWaveSpeed` holds maximum wave speed
- updated during computation of numerical fluxes
in method `computeNumericalFluxes()`:
$$\text{maxTimestep} = \text{std::min}(\text{dx}/\text{maxWaveSpeed}, \text{dy}/\text{maxWaveSpeed});$$
- simple “getter” method defined in class `SWE_Block`:
float `getMaxTimestep()` { **return** `maxTimestep`; };
- hence: `getMaxTimestep()` for current time step should be called
after `computeNumericalFluxes()`

Update Unknowns – Euler Time Stepping

```
for(int i=1; i < nx+1; i++) {  
    for(int j=1; j < ny+1; j++) {  
        h[i][j] -= dt/dx * (hNetUpdatesRight[i-1][j-1]  
                           + hNetUpdatesLeft[i][j-1] )  
        + dt/dy * (hNetUpdatesAbove[i-1][j-1]  
                  + hNetUpdatesBelow[i-1][j] );  
        hu[i][j] -= dt/dx * (huNetUpdatesRight[i-1][j-1]  
                             + huNetUpdatesLeft[i][j-1] );  
        hv[i][j] -= dt/dy * (hvNetUpdatesAbove[i-1][j-1]  
                             + hvNetUpdatesBelow[i-1][j] );  
        /* ... */  
    }  
}
```

(cmp. file `SWE_WavePropagationBlock.cpp`)

Goals for (Parallel) Implementation

Spatial Discretization:

- allow different parallel programming models
- and also to switch between different numerical models

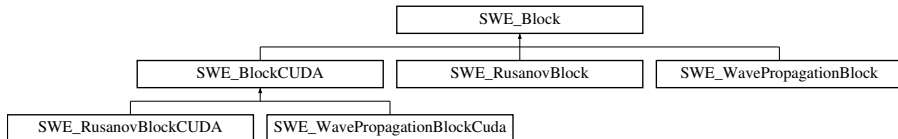
⇒ **class hierarchy of numerical vs. programming models**

Hybrid Parallelization:

- support two levels of parallelization
- coarse-grain parallelism across Cartesian grid patches
- fine-grain parallelism on patch-local loops

⇒ **separate fine-grain and coarse-grain parallelism**
(plug&play principle)

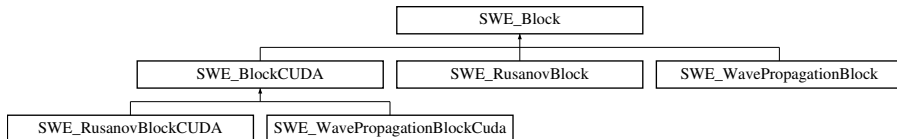
SWE Class Design



abstract class **SWE_Block**:

- base class to hold data structures (arrays h , h_u , h_v , b)
- manipulate ghost layers
- methods for initialization, writing output, etc.
- defines interface for main time-stepping loop:
computeNumericalFluxes(), updateUnknowns(), ...

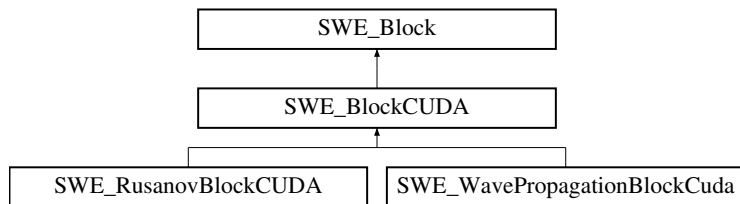
SWE Class Design (2)



derived classes:

- for different model variants: `SWE_RusanovBlock`, `SWE_WavePropagationBlock`, ...
- for different programming models: `SWE_BlockCUDA`, `SWE_BlockArBB`, ...
- override `computeNumericalFluxes()`, `updateUnknowns()`, ...
→ methods relevant for parallelization

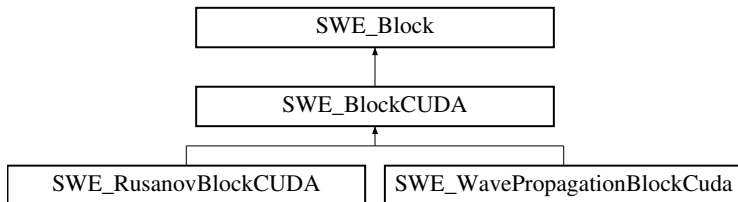
SWE Class Design – SWE_BlockCUDA



abstract class `SWE_Block`:

- base class to hold data structures (arrays `h`, `hu`, `hv`, `b`)
- manipulate ghost layers
- methods for initialization, writing output, etc.

SWE Class Design – SWE_BlockCUDA (2)



derived classes:

- for different model variants: `SWE_RusanovBlock`, `SWE_WavePropagationBlock`, ...
- for different programming models: `SWE_BlockCUDA`, `SWE_BlockArBB`, ...
- override `computeNumericalFluxes()`, `updateUnknowns()`, ...
→ methods relevant for parallelization

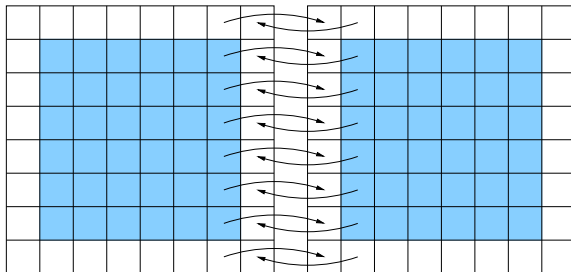
Example: SWE_WavePropagationBlockCUDA

```
class SWE_WavePropagationBlockCuda: public SWE_BlockCUDA {  
    /*— definition of member variables skipped —*/  
    public:  
        // compute a single time step (net-updates + update of the cells).  
        void simulateTimestep( float i_dT );  
        // simulate multiple time steps ( start and end time provided as param  
        float simulate(float, float);  
        // compute the numerical fluxes (net-update formulation here).  
        void computeNumericalFluxes();  
        // compute the new cell values.  
        void updateUnknowns(const float i_deltaT);  
};
```

(in file SWE_WavePropagationBlockCuda.hh)

Part III

Parallel Programming Patterns



Computing the Net Updates

Parallel Programming Patterns

- compute net updates on left/right edges:

```
for(int i=1; i < nx+2; i++) in parallel {
    for(int j=1; j < ny+1; j++) in parallel {
        float maxEdgeSpeed;
        fWaveComputeNetUpdates( 9.81,
                                h[i-1][j], h[i][j], hu[i-1][j], hu[i][j], /* ... */ );
    }
}
```

- compute net updates on top/bottom edges:

```
for(int i=1; i < nx+1; i++) in parallel {
    for(int j=1; j < ny+2; j++) in parallel {
        fWaveComputeNetUpdates( 9.81,
                                h[i][j-1], h[i][j], hv[i][j-1], hv[i][j], /* ... */ );
    }
} (function fWaveComputeNetUpdates() defined in file solver/FWaveCuda.h)
```


Computing the Net Updates

Options for Parallelism

Parallelization of computations:

- compute all vertical edges in parallel
- compute all horizontal edges in parallel
- compute vertical & horizontal edges in parallel (task parallelism)

Parallel access to memory:

- concurrent read to variables h , h_u , h_v
- exclusive write access to net-update variables on edges

Updating the Unknowns

Parallel Programming Patterns

- update unknowns from net updates on edges:

```

for(int i=1; i < nx+1; i++) in parallel {
    for(int j=1; j < ny+1; j++) in parallel {
        h[i][j] -= dt/dx * (hNetUpdatesRight[i-1][j-1]
                           + hNetUpdatesLeft[i][j-1])
               + dt/dy * (hNetUpdatesAbove[i-1][j-1]
                           + hNetUpdatesBelow[i-1][j]);
        hu[i][j] -= dt/dx * (huNetUpdatesRight[i-1][j-1]
                             + huNetUpdatesLeft[i][j-1]);

        /* ... */
    }
}

```

Updating the Unknowns

Options for Parallelism

Parallelization of computations:

- compute all cells in parallel

Parallel access to memory:

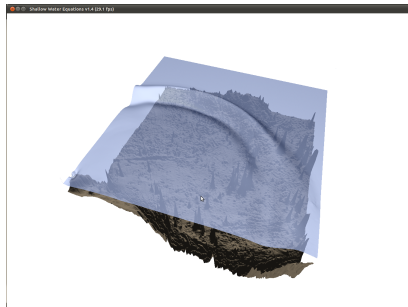
- concurrent read to net-updates on edges
- exclusive write access to variables h , h_u , h_v

“Vectorization property”:

- exactly the same code for all cell!

Part IV

SWE and CUDA



SWE_BlockCUDA – GPU Memory

Additional Member Variables in class SWE_BlockCUDA:

- base class to hold data structures (arrays h , h_u , h_v , b)
- manipulate ghost layers
- methods for initialization, writing output, etc.

Allocate unknowns h , h_u , h_v , b in SWE_BlockCUDA:

```
int size = (nx+2)*(ny+2)*sizeof(float);  
// allocate CUDA memory for unknowns h, h_u, h_v and bathymetry b  
cudaMalloc((void**)&hd, size);  
cudaMalloc((void**)&hud, size);  
cudaMalloc((void**)&hvd, size);  
cudaMalloc((void**)&bd, size);
```

(see constructor SWE_BlockCUDA(...) in file SWE_BlockCUDA.cu)

SWE_BlockCUDA – GPU Memory (2)

Define & Allocate Member Variables in SWE_BlockCUDA:

```
SWE_BlockCUDA::SWE_BlockCUDA(/*-- parameters--*/)
: SWE_Block(_offsetX,_offsetY)
{ /*-- further initializations skipped --*/
    int size = (nx+2)*(ny+2)*sizeof(float);
    // allocate CUDA memory for unknowns h,hu,hv and bathymetry b
    cudaMalloc((void**)&hd, size);
    checkCUDAError("allocate device memory for h");
    cudaMalloc((void**)&hud, size);
    checkCUDAError("allocate device memory for hu");
    cudaMalloc((void**)&hvd, size);
    checkCUDAError("allocate device memory for hv");
    cudaMalloc((void**)&bd, size);
    checkCUDAError("allocate device memory for bd");
    /*-- allocation of ghost/copy layer to follow --*/
}
```

(see file SWE_BlockCUDA.cu)

Excursion: Checking for CUDA Errors

- CUDA API functions typically return error code as value
- but no exceptions, (immediate) crashes, etc.
- error code should thus be checked after each function call

⇒ helper function defined in SWE_BlockCUDA:

```
void checkCUDAError(const char *msg)
{
    cudaError_t err = cudaGetLastError();
    if ( cudaSuccess != err)
    {
        fprintf ( stderr , "\nCuda error (%s): %s.\n",
                  msg, cudaGetErrorString( err) );
        exit(-1);
    }
}
```

(see file SWE_BlockCUDA.cu)

SWE_BlockCUDA – Synchronize Memory

Methods to copy CPU memory to GPU memory:

- called after each external write to arrays h, hu, hv, b (read data from file, set initial conditions, etc.)
- allows to implement individual methods on GPU
- SWE allows data in main memory to be not up-to-date (goal: perform simulation entirely on GPU)

Interface defined in class SWE_Block:

```
void SWE_Block::synchAfterWrite() {  
    synchWaterHeightAfterWrite();  
    synchDischargeAfterWrite();  
    synchBathymetryAfterWrite();  
}
```

(see file SWE_Block.cpp)

CUDA Example: Synchronize Water Height

Method `synchWaterHeightAfterWrite()`:

- synchronize array `h` on CPU and GPU memory
- **after an external update of the water height `h`**
(i.e., after an update of CPU main memory)
- copies entire array `h` (incl. ghost layers) into array `hd`

```
void SWE_BlockCUDA::synchWaterHeightAfterWrite() {  
    /*— — —*/  
    int size = (nx+2)*(ny+2)*sizeof(float);  
    cudaMemcpy(hd,h.elemVector(), size, cudaMemcpyHostToDevice);  
    checkCUDAError("memory of h not transferred");  
}
```

(see file `SWE_BlockCUDA.cu`)

SWE_BlockCUDA – Synchronize Memory (2)

Methods to copy GPU memory to CPU memory:

- called before each external output of arrays h, hu, hv, b (write output to file, etc.)
- allows to implement individual methods on GPU
- helpful for debugging

Interface defined in class SWE_Block:

```
void SWE_Block::synchBeforeRead() {  
    synchWaterHeightBeforeRead();  
    synchDischargeBeforeRead();  
    synchBathymetryBeforeRead();  
}
```

(see file SWE_Block.cpp)

CUDA Example: Synchronize Water Height

Method `synchWaterHeightBeforeRead()`:

- synchronize array `h` on GPU and CPU memory
- **after an update of the water height `hd` on the GPU**
(e.g., after computation of one or more time steps on the GPU)
- copies entire array `hd` (incl. ghost layers) into array `h`

```
void SWE_BlockCUDA::synchWaterHeightBeforeRead() {  
    /*--- */  
    int size = (nx+2)*(ny+2)*sizeof(float);  
    cudaMemcpy(h.elemVector(),hd, size, cudaMemcpyDeviceToHost);  
    checkCUDAError("memory of h not transferred");  
    /*--- */  
}
```

(see file `SWE_BlockCUDA.cu`)

CUDA Parallelization – Afternoon Session 1

Goal: “run everything on the GPU” → remember main loop:

```
while( t < ... ) {  
    // set boundary conditions  
    splash.setGhostLayer();  
  
    // compute fluxes on each edge  
    splash.computeNumericalFluxes();  
  
    // set largest allowed time step:  
    dt = splash.getMaxTimestep();  
    t += dt;  
  
    // update unknowns in each cell  
    splash.updateUnknowns(dt);  
};
```

CUDA: Set Ghost Layer

Implementation in `SWE_Block::setGhostLayer()`:

1. call `setBoundaryConditions()`
→ set simple, block-local boundary conditions (“real boundaries”)
2. transfer data between ghost and copy layers
→ to be discussed in more detail (later)

```
void SWE_BlockCUDA::setBoundaryConditions() {  
    /*-- some code skipped --*/  
    if (boundary[BND_LEFT] == PASSIVE || /*-- --*/) {  
        /*-- --*/  
    }  
    else {  
        dim3 dimBlock(1,TILE_SIZE);  
        dim3 dimGrid(1,ny/TILE_SIZE);  
        kernelLeftBoundary<<<dimGrid,dimBlock>>>(  
            hd,hud,hvd,nx,ny,boundary[BND_LEFT]);  
    }  
};
```

(see file `SWE_BlockCUDA.cu`)

CUDA: Set (Simple) Boundary Conditions

```
--global--  
void kernelLeftBoundary(float* hd, float* hud, float* hvd,  
                        int nx, int ny, BoundaryType bound)  
{  
    // determine j coordinate of current ghost cell :  
    int j = 1 + TILE_SIZE*blockIdx.y + threadIdx.y;  
    // determine position of ghost and copy cell in array:  
    int ghost = getCellCoord(0,j,ny);  
    int inner = getCellCoord(1,j,ny);  
  
    // consider only WALL & OUTFLOW boundary conditions:  
    hd[ghost] = hd[inner];  
    hud[ghost] = (bound==WALL) ? -hud[inner] : hud[inner];  
    hvd[ghost] = hvd[inner];  
}
```

(in file SWE_BlockCUDA_kernels.cu)

CUDA Parallelization – Afternoon Session 1

Goal: “run everything on the GPU”

⇒ **functions and kernels to implement:**

- compute fluxes on each edge:

→ **splash.computeNumericalFluxes();**

```
dim3 dimBlock(TILE_SIZE,TILE_SIZE);  
dim3 dimGrid(nx/TILE_SIZE,ny/TILE_SIZE);  
computeNetUpdatesKernel<<<dimGrid,dimBlock>>>(  
    hd, hud, hvd, bd, /* ... */ , nx,ny);
```

- update unknowns in each cell:

→ **splash.updateUnknowns(dt);**

```
dim3 dimBlock(TILE_SIZE, TILE_SIZE);  
dim3 dimGrid(nx/TILE_SIZE, ny/TILE_SIZE);  
updateUnknownsKernel<<<dimGrid,dimBlock>>>(  
    hd, hud, hvd, /* ... */ , nx, ny, dt, 1.0f/dx, 1.0f/dy);
```

CUDA Parallelization – Afternoon Session 1

Roadmap:

1. replace example solution by code template:
→ in SWE “home” directory:

```
cp src_skeleton/* src/
```

2. two CUDA kernels to be implemented
in the file SWE_WavePropagationBlockCuda_kernels.cu:

```
void computeNetUpdatesKernel([...])  
void updateUnknownsKernel([...])
```

3. **hint:** first use the following pattern:
→ transfer variables h, hu, hv to GPU memory
→ call to CUDA kernel
→ transfer updated variables back to CPU memory

CUDA Parallelization – Afternoon Session 1

Goal: “run *really(?)* everything on the GPU”

- focus on computation of net updates and Euler time step, first
- missing: set largest allowed time step
→ **splash.getMaxTimestep();**
- requires computation of a maximum/minimum (CFL condition: maximum wave speed required)
→ best done in kernel for net updates
- will be left for session 2 (or even 3)
→ use fixed time step until then . . .

```
// update unknowns in each cell  
splash.updateUnknowns(dt);
```

- set dt to some good value
- or trust method computeMaxTimestep() in class SWE_Block

Part V

Optimization of the SWE-CUDA Kernels

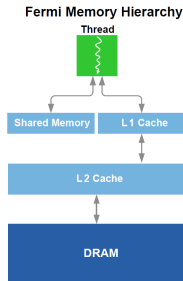


image: NVIDIA

SWE-CUDA – Memory-Bound Performance

A performance estimate for SWE:

- assumption: performance is **memory-bound**
- presentation laptop has a bandwidth (GPU main memory) of 11.2 GB/s
- what is the possible performance of the SWE code?

Memory transfer in SWE:

- consider mesh of size 1024×1024 , thus 1 M<cells>
- variables h, h_u, h_v, b : 4×4 bytes, thus 16 MB
- net updates: 4×4 bytes per edge, thus 32 MB
- how many read & write accesses in each kernel?

SWE-CUDA – Memory-Bound Performance (2)

Memory accesses in computeNetUpdates:

- read variables h , h_u , h_v , b : 16 MB
- write netUpdates: 32 MB

Memory accesses in updateUnknowns:

- read netUpdates: 32 MB
- write variables h , h_u , h_v : 12 MB

Total memory transfer:

- neglect computation of maximum wave speed
- read 48 MB, write 44 MB per time step
- $11.2 \text{ GB/s} \approx 120 \text{ time steps per sec.}$

SWE-CUDA – Memory-Bound Performance (3)

Road blocks for memory-bound performance:

- assumed that each kernels reads/writes any piece of data only once
- currently not the case for read accesses

Read accesses in computeNetUpdates:

- each cell reads h , h_u , h_v , b from left/bottom and right/top cell
→ doubles number of read accesses
- kernel is called twice (left/right and bottom/top updates)
→ doubles number of read accesses
- new value: read 192 MB, write 44 MB per time step
→ $11.2 \text{ GB/s} \approx 60 \text{ time steps per sec.}$?

Read accesses in updateUnknowns:

- actually no extra read or write accesses

CUDA Parallelization – Level 2

Optimization of kernels:

- coalesced access to GPU memory
- use of shared memory and registers

```
__shared__ float Fds[TILE_SIZE+1][TILE_SIZE+1];  
__shared__ float Gds[TILE_SIZE+1][TILE_SIZE+1];  
/* ... */  
int iEdge = getEdgeCoord(i,j,ny); // index of right/top Edge  
Fds[tx+1][ty] = Fhd[iEdge];  
Gds[tx][ty+1] = Ghd[iEdge];  
/* ... */  
h = hd[iElem] - dt * ( (Fds[tx+1][ty] - Fds[tx][ty]) * dxi  
                      + (Gds[tx][ty+1] - Gds[tx][ty]) * dyi );
```

(in file SWE_RusanovBlockCUDA_kernels.cu)

Maximum Wave Speeds

Parallel Reduction Revisited

Computation of “Net Updates”:

- kernel computes wave speeds for every edge/cell
- also required to compute the CFL condition
→ parallel maximum computation required

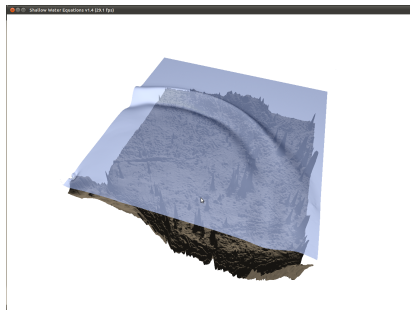
Optimization approach:

- keep wave speeds in shared memory
- compute maximum wave speed of a tile in shared memory
- subsequent parallel reduction only on tile-maxima

Some Aspects of CUDA Parallelization

Level 3: more advanced optimizations

- “kernel fusion”: merge computation of fluxes with updates of unknowns
- merge maximum-reduction on wave speeds (for CFL condition) with flux computation (or update of velocities)
- allows interactive/“real-time” simulation (800×800 cells)



Net Updates and Updating Unknowns

Parallel Programming Patterns Revisited

Anticipate new parallel program:

For each cell in parallel(!) compute:

1. net updates for all edges (vertical & horizontal)
2. update cell unknowns from net updates
write to next-timestep copies of h , h_u , h_v !

Parallel access to memory:

1. concurrent read to h , h_u , h_v ; exclusive write to net updates
 2. concurrent read to net updates; exclusive read to h , h_u , h_v
- ⇒ 2 after 1 for all cells, so everything is fine?
- ⇒ **unfortunately not!** (consider CUDA blocks, warps, etc.)
- ⇒ **may be cured:** old/new copy for h , h_u , h_v

Performance Contest

SWE on a Tesla C2070 (mathgpu)

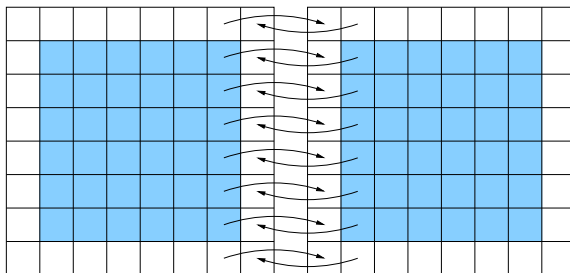
- 448 stream processing units
- memory bandwidth: 97.6 GB/s (acc. to bandwidth test)
- theoretical peak performance: ≈ 1 TFlop/s

How much do you get?

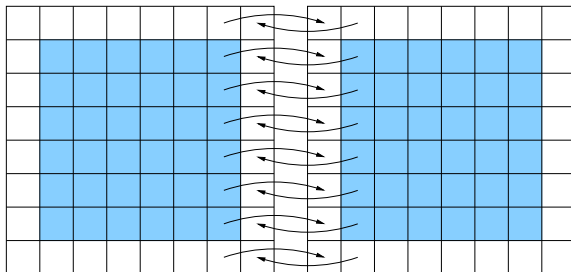
- in terms of memory throughput?
- in terms of Flop/s?
- in terms of processed cells per second?

Part VI

Parallelization on Hybrid Architectures



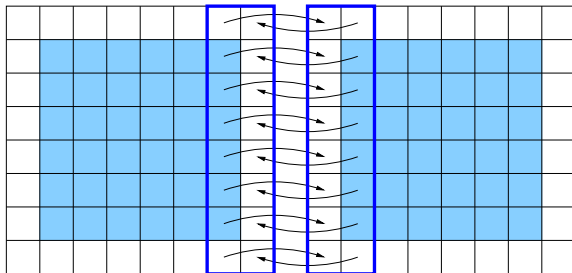
Exchange of Values in Ghost/Copy Layers



Straightforward Approach:

- boundary conditions OUTFLOW, WALL vs. CONNECT or PARALLEL
- disadvantage: method `setGhostLayer()` needs to be implemented for each derived class

Exchange of Values in Ghost/Copy Layers (2)



Implemented via Proxy Objects:

- `grabGhostLayer()` to write into ghost layer
- `registerCopyLayer()` to read from copy layer
- return proxy object (class `SWE_Block1D`) that references one row/column of the grid

SWE_BlockCUDA – Update of Ghost Layers

Memory-Synchronization Revisited

- ghost layers might be updated in each time step
→ conditions PASSIVE, CONNECT
- updated ghost layers in CPU memory need to be copied to GPU

```
void SWE_BlockCUDA::synchGhostLayerAfterWrite() {  
    if (boundary[BND_LEFT] == PASSIVE ||  
        boundary[BND_LEFT] == CONNECT) {  
        // transfer h, hu, hv from left ghost layer to resp. device mem  
        cudaMemcpy(hd, h[0], (ny+2)*sizeof(float),  
                   cudaMemcpyHostToDevice);  
        /*— same for hud/hu and hvd/hv —*/  
    }  
};
```

(in file SWE_BlockCUDA.cu)

SWE_BlockCUDA – Update of Copy Layers

Memory-Synchronization Revisited

- copy layers need to be updated in each time step
→ conditions PASSIVE, CONNECT
- requires transfer from GPU to CPU memory

```

void SWE_BlockCUDA::synchCopyLayerBeforeRead() {
    /*-- left and right copy layer skipped --*/
    int size = 3*(nx+2);
    // bottom copy layer
    if (... || boundary[BND_BOTTOM] == CONNECT) {
        dim3 dimBlock(TILE_SIZE,1);
        dim3 dimGrid(nx/TILE_SIZE,1);
        kernelBottomCopyLayer<<<dimGrid,dimBlock>>>(
            hd,hd,hvd,bottomLayerDevice+size,nx,ny);
        cudaMemcpy(bottomLayer+size, bottomLayerDevice+size,
                    size*sizeof(float), cudaMemcpyDeviceToHost);
    };
    /*-- ... --*/
  
```

(in file SWE_BlockCUDA.cu)

MPI Parallelization

– Exchange of Ghost/Copy Layers

```
SWE_Block1D* leftInflow = splash.grabGhostLayer(BND_LEFT);  
SWE_Block1D* leftOutflow = splash.registerCopyLayer(BND_LEFT);
```

```
SWE_Block1D* rightInflow = splash.grabGhostLayer(BND_RIGHT);  
SWE_Block1D* rightOutflow = splash.registerCopyLayer(BND_RIGHT);
```

```
MPI_Sendrecv(leftOutflow->h.elemVector(), 1, MPI_COL, leftRank, 1,  
             rightInflow->h.elemVector(), 1, MPI_COL, rightRank, 1,  
             MPI_COMM_WORLD,&status);
```

```
MPI_Sendrecv(rightOutflow->h.elemVector(), 1, MPI_COL, rightRank, 4,  
             leftInflow->h.elemVector(), 1, MPI_COL, leftRank, 4,  
             MPI_COMM_WORLD,&status);
```

(cmp. file `examples/swe_mpi.cpp`)

Teaching Parallel Programming with SWE

SWE in Lectures, Tutorials, Lab Courses:

- non-trivial example, but model & implementation easy to grasp
- allows different parallel programming models (MPI, OpenMP, CUDA, Intel TBB/ArBB, OpenCL, ...)
- prepared for hybrid parallelisation

Some Extensions:

- ASAGI - parallel server for geoinformation (S. Rettenberger, Master's thesis)
- OpenGL real-time visualisation of results (T. Schnabel, student project)

→ <http://www5.in.tum.de/SWE/>

→ <https://github.com/TUM-I5>



References/Literature

- George, D. L. (2008), *Augmented Riemann solvers for the shallow water equations over variable topography with steady states and inundation*. J. Comput. Phys. 227 (6), p. 3089–3113
- Bale, D. S. (2002), R. J. LeVeque, S. Mitran, and J. A. Rossmannith, *A wave-propagation method for conservation laws with spatially varying flux functions*. SIAM J. Sci. Comput. 24, p. 955–978.
- M. Bader (2012) and A. Breuer: *Teaching Parallel Programming Models on a Shallow-Water Code*. In: 11th Int. Symp. on Parallell. and Dist. Computing (ISPDC 2012). IEEE Computer Society.